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A REVIEW OF STUDIES ON MATHEMATICAL THINKING IN TURKEY

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Abstract

This study aims at reviewing the tendencies of the studies on mathematics education in Turkey with particular regard to mathematical thinking. In this respect, the theses in the National Center for Theses and Dissertations and the journals published online at the DergiPark (The most comprehensive collection of academic journals in Turkey) portal were searched and 48 studies on mathematical thinking were found. In this qualitative study, the studies reviewed were analyzed descriptively. In the light of the findings obtained, it is found that there are 23 articles, 12 master's theses and 13 doctoral theses. It is also found that while these studies were limited in number until 2010, the number has increased after 2010. When the studies are examined regarding their methodology, it is seen that qualitative researches are in majority; however, there

are also quantitative and mixed studies. In addition to this finding, it is also observed that generally open-ended questions, interviews, observations and video recordings are used in qualitative studies, and content analysis is preferred. In the quantitative studies, on the other hand, it is seen that scales are generally preferred as data collection tools. In addition, it is observed that the researchers generally selected middle school students and elementary pre-service mathematics teachers as their sample. The sample size is mostly between 11 and 30 and generally, 201 to 300 samples are determined. According to the results obtained, it is seen that mathematical thinking has become a popular subject recently. Therefore, it is thought that conducting more studies on mathematical thinking and preferring different sample groups would be beneficial for improving mathematics education.

Keywords: Mathematics Education, Mathematical Thinking, Content Analysis.

INTRODUCTION

The changing world requires individuals, who are well aware of themselves and their environment and who know how and in what way to think. The path of raising such individuals passes through new conceptions of education that aims at upskilling individuals with analyzing certain structures, seeing the relations inside the structures and forming cause and effect relations between events, i.e. reasoning (Umay, 2003). Concordantly, when it is considered that thinking is the most prominent feature that distinguishes humans from other living things, it is true that expurgated, simple and genuine thought, sound and timely produced, would set an individual active in their environment. Because, following this, the individuals accommodate themselves to the society they live in, and take an active role in its development (Alkan & Bukova Güzel, 2005).

When the properties of mathematics are considered, it is seen that mathematics is one of the tools required for upskilling children and youngsters with knowledge and skills required by daily life, teaching them to solve problems, enabling them with thought patterns within the problem solving approach and preparing them for the future (Yıldırım, 2006).

Mathematics is one of the most significant tools that is known to improve thinking. As it is known, the basic feature that separates humans from other living things is thinking, i.e. the ability of making sense of the events and reorganizing the circumstances suited for themselves. Thus, mathematics education comprises one of the important, probably the most important, building blocks of basic education (Umay, 2003). It is a commonly held view that learning mathematical thinking provides mastery in most areas of an individual's life. The most significant feature of becoming skillful in mathematical thinking is that it enables the individual both to improve the mathematical innovative thinking and productive problem solving skills, and to gain an astounding self-confidence (Özer, 2005).

Mathematical thinking can be considered as the direct or indirect use of mathematical knowledge, concepts and processes in solving problems (Henderson, Baldwin, Dasigi, Dupras, Fritz, Ginat, Goelman, Hamer, Hitchner, Lloyd, Marion, Riedsel, Walker, 2001). Put it differently, mathematical thinking is the explicit or inexplicit use of mathematical methods and techniques in solving problems (Henderson, 2002). Individuals, in every stage of their lives, use mathematical thinking, consciously or unconsciously, to solve the issues they encounter. So mathematical thinking is a pattern of thought used not by the mathematicians only but all the people during their whole lives (Bilitzer, 2003).

Mathematical thinking skill and the use of mathematical thinking in problem solving, has become an important objective for the schools. In this respect, mathematical thinking has come to the fore in supporting the enhancement of science, technology, economic life and economic development

(Stacey, 2006). Similarly, NCTM (2000) lays emphasis on the fact that the need for understanding and using mathematics in myriad areas of life, and thus mathematical thinking and problem solving skills should be improved. In this context, some research has been carried out for mathematical thinking (Kocaman, 2017; Liu, 2014; Nabb, 2013; Olgun, 2016; Soto; 2014; Yıldırım, 2015). For this purpose, several studies were conducted to review the research aiming at increasing and improving the mathematics levels of the students. At the same time, various studies have been done to evaluate the studies to increase and improve the mathematics levels of the students (Baki, Güven, Karataş, Akkan & Çakıroğlu 2011; Çiltaş, Güler & Özbilir, 2012; Ulutaş & Ubuz, 2008).

Reviewing studies in the mathematics education field provides the researchers with information about the topics studied. In addition, the review of variables such as the methods used in these studies, the type of the sample mass, the sample size, data collection tools and data analysis methods are of great significance for the studies to be conducted in the future. This study aimed at reviewing the studies in mathematics education field with regard to various variables within the perspective of "mathematical thinking". In this respect, answers for the following questions were sought: Considering the studies on mathematical thinking conducted in Turkey;

1. What is their distribution with regard to publication type (article, master's thesis, doctoral thesis)?
2. What is their distribution with regard to years?
3. What is their distribution with regard to methods used?
4. What is their distribution with regard to sample types?
5. What is their distribution with regard to sample size?
6. What is their distribution with regard to data collection tools used?
7. What is their distribution with regard to the number of data collection tools used?

8. What is their distribution with regard to data analysis methods used?

METHOD

The Model used in the Study

A qualitative study was conducted in accordance with the aim of this research. Qualitative research is defined as a research in which qualitative data collection tools such as observation, interview and document analysis are used, and in which a qualitative process is followed for revealing the perceptions and events realistically and holistically in their natural environment (Demirbaş, 2014).

Data Collection

The studies about "mathematical thinking" were searched among the studies in mathematics education field conducted in Turkey. In this respect, the journals indexed by DergiPark (The most comprehensive collection of academic journals in Turkey) and the master's and doctoral theses in the National Center for Theses and Dissertations affiliated with the Council of Higher Education were searched. At the end of the research, 48 studies related to "mathematical thinking" were accessed. These studies were analyzed in line with the sub-problems of this study.

Data Analysis

Descriptive analysis is used in analyzing the studies obtained at the end of the search. The purpose of descriptive analysis is to present the findings to the reader in an ordered and interpreted manner. With this purpose, the data obtained are first described in an explicit and systematic way. Later, these descriptions are elaborated and interpreted, the cause and effect relation is addressed, and certain results are obtained (Yıldırım & Şimşek, 2011). By which categories the studies accessed would be reviewed was determined in line with expert views. In this respect, the studies reviewed were analyzed in terms of publication type,

publication year, the research method used, sample type, sample size, data collection tools, number of data collection tools and data analysis methods used.

The studies accessed were categorized as article, master's thesis and doctoral thesis with regard to publication type. In the publication year category their distribution with regard to publication dates was given. The methods used in the studies were grouped as qualitative, quantitative and mixed. The sample types used in the studies were grouped under common themes and their sizes were grouped within certain ranges. The data collection tools, their number and the data analysis methods were obtained by the classification of the data collection tools used in the studies. The analysis of the studies was conducted by the researchers independently and the level of concordance between the themes provided by the researchers was found 95%. The data obtained were digitized and the results were descriptively presented as frequency and percentage tables.

FINDINGS

The data of the study were analyzed considering the research questions. At the end of the analyses, the following findings were obtained.

Distribution of Studies with Regard to Publication Type

When the publication type of the studies was examined, their frequency and percentage distributions were found as presented below.

Table 1. Distribution of studies by publication type

Publication Type	Frequency (f)	Percent (%)
Article	23	48
Master Thesis	12	25
Doctoral Thesis	13	27
Total	48	100

When the Table is examined, it is seen that articles are the most frequently seen publications with a total number of 23 among the accessed studies. It is also found that there are 12 master's theses and 13 doctoral theses. These data indicate that almost half of the studies are articles (48%) and then number of master's theses (25%) and doctoral theses (27%) are similar.

Distribution of Studies with Regard to Publication Year

When the publication years of the accessed studies are examined, the following statistics are obtained.

Table 2. Distribution of studies by years

Years	1992	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018
Frequency (f)	1	1	2	2	1	2	1	1	4	2	7	3	6	4	4	7
Percent (%)	2,1	2,1	4,2	4,2	2,1	4,2	2,1	2,1	8,3	4,2	14,5	6,3	12,5	8,3	8,3	14,5

When the studies on mathematical thinking are examined with regard to their publication years, it is found that the first study was published in 1992. It is also seen that the following studies were made after 2004. However, the studies conducted after 2004 and until 2010 were limited in number. It is found that the number of the studies on this topic had increased after 2010 such that the frequency of the studies increased in 2013 and it is found that seven studies were conducted in 2013. However, it is found that there was a decrease in the number studies in 2014 and the studies conducted were reduced to three in 2014. Another increase is observed in 2015. It is found that in 2018 the numbers of the studies conducted are equal to the number of the studies in 2013.

Research Methods Used in the Studies

At the end of the analysis of the research methods used in the reviewed studies, the findings presented in the following table are found.

Table 3. Distribution of studies according to research methods

Method	Frequency (f)	Percent (%)
Quantitative	18	37,5
Qualitative	21	43,75
Mixed	9	18,75
Total	48	100

When the research methods frequently used in studies on mathematical thinking are examined, it is seen that qualitative research methods are preferred in 21 studies. It is found that this number equals to almost half of the studies with 43.75%. In addition, it is observed that quantitative research methods are used in 18 studies (37.5%) and mixed research methods are used in nine studies (18.75%).

Sample Type Used in the Studies

At the end of the analyses of sample types used in the reviewed studies, the findings presented in the following table are found.

Table 4. Distribution of studies by sample type

Sample Type	Frequency (f)	Percent (%)
Middle School Students	14	23,7
Elementary Pre-service Mathematic Teachers	12	20,3

Secondary Pre-service Mathematic Teachers	9	15,3
High School Students	8	13,5
Pre-serice Class Teacher	5	8,5
Mathematics Teachers	5	8,5
Business Students	2	3,4
Primary School Students	1	1,7
Academician	1	1,7
Pre-service Science Teachers	1	1,7
Preschool Students	1	1,7
Total	59	100

When the Table is examined, it is seen that the researchers generally used middle school students (23.7%) and elementary pre-service mathematics teachers (20.3%) as samples in the studies they conducted on mathematical thinking. In other words, it is found that the researchers conducted research on secondary education. It is also found that there is a considerable number of studies, which determined secondary pre-service school mathematics teachers (15.3%) and high school students (13.5%) as samples. On the other hand, it is found that there are very few studies conducted with the participation of elementary school students, academicians, pre-service science teachers, and preschoolers.

Sample Size Used in the Studies

At the end of the analyses of sample sizes used in the reviewed studies, the findings presented in the following table are found.

Table 5. Distribution of studies according to sample size

Sample Size	Frequency (f)	Percent (%)
1-10 inter	7	14,6
11-30 inter	9	18,7
31-50 inter	7	14,6
51-100 inter	6	12,5
101-200 inter	5	10,3
201-300 inter	8	16,7
301-500 inter	2	4,2
501-1000 inter	2	4,2
1001 and over	2	4,2
Total	48	100

When the sample sizes in the studies are examined, it is seen that generally sample sizes comprising 11 to 30 individuals (18.7%) and 201 to 300 individuals (16.7%) are preferred. In the studies reviewed, it is found that the rate of preferring sample sizes of 1-10 and 31-50 is 14.6%, of 51-100 is 12.5%, and of 101-200 is 16.7%. It is seen that the researchers determined their sample sizes generally as 11-30 and 201-300 ranges, and the rate of determining sample sizes larger than 300 is very low.

Data Collections Tools Used in the Studies

At the end of the analyses of data collection tools used in the reviewed studies, the findings presented in the following table are found.

Table 6. Distribution of studies according to data collection tools

Data Collection Tools	Frequency (f)	Percent (%)
Open-ended Question	22	26,5

Scale	19	22,9
Interview Form	10	12
Observation	9	10,8
Worksheet	7	8,5
Achievement test	7	8,5
Video Recordings	5	6
Rubrics	3	3,6
Written Document	1	1,2
Total	83	100

When the table is examined, it is seen that the researchers use the open-ended questions the most. Next, they used scales (22.9%) as data collection tools. In addition to these, it is found interview forms (12%) and observations (9%) are used frequently. It is observed that data collection tools such as worksheets (7%), achievement tests (7%) and video recordings (5%) are also used. It is understood that data collection tools such as rubrics (3.6%) and written documents (1.2%) are not used quite frequently. With regard to this data, it is found that the researchers use the qualitative data collection tools frequently in their studies on mathematical thinking.

Number of Data Collection Tools Used in the Studies

At the end of the analyses of the number data collection tools used in the reviewed studies, the findings presented in the following table are found.

Table 7. Distribution of studies by number of data collection tools

Number of Data Collection Tools	1	2	3	5	6	Total
Frequency (f)	21	20	4	2	1	48

Percent (%)	43,7	41,7	8,3	4,2	2,1	100

When the number of data collection tools used in the studies on mathematical thinking, it is seen that the great majority of the studies use one data collection tool (43.7%). It is also observed that the rate of the studies, in which two data collection tools are used, is 41.7%. These data indicate that generally one or two data collection tools are used in the studies on mathematical thinking. In addition, it is observed that there are studies, in which three, five and six data collection tools are used, despite being infrequent.

Data Analysis Methods Used in the Studies

At the end of the analyses of the analysis methods used in the reviewed studies, the findings presented in the following table are found.

Table 8. Distribution of studies according to data analysis methods

Data Analysis Methods	Frequency (f)	Percent (%)
Content Analysis	19	20,2
t-test	14	14,8
Frequency	11	11,7
Percent	11	11,7
Descriptive Analysis	10	10,6
Correlation	6	6,3
Arithmetic Mean	5	5,3
Standard Deviation	4	4,3
Factor Analysis	4	4,3
ANOVA	3	3,2
Regression	2	2,1
Kolmogrov Smirnov	1	1,1

MANCOVA	1	1,1
Thematic	1	1,1
Fenomonolojik	1	1,1
Man Whitney U	1	1,1
Total	94	100

When the data analysis methods used in the studies are examined, it is seen that content analysis is used in 20.2% of the studies, t-test in 14.8%, frequency and percentage in 11.7% of the studies. In addition, the rate of descriptive analysis use is 10.6% and the rate for correlation is 6.3%. These data indicate that the researchers mostly preferred qualitative data analysis methods and the frequency of quantitative data analysis methods is low.

DISCUSSION

In this study, the research on mathematical thinking in Turkey was reviewed, and the findings obtained in line with the sub-questions of the study were interpreted. When the studies accessed at the end of the surveys are reviewed with regard to years, it is found that very few studies had been conducted until 2010, and the number of the studies has increased after 2010. This shows that mathematical thinking has becoming a prominent subject recently. The importance of students' mathematical thinking in their daily lives is expressed in several studies (Blitzer, 2003, Lim & Hwa, 2006; Schoenfeld, 1992; Tall, 1995). Therefore, it is considered that increasing the number of studies on mathematical thinking would be beneficial both in mathematics education and in daily lives of individuals.

When the research methods frequently used in studies on mathematical thinking are examined, it is seen that qualitative research methods are used in 43.75% of the studies. In addition, it is observed that quantitative research methods are used in 37.5% of the studies and mixed research methods in 18.75%

of them. Çiltaş, Güler and Sözbilir (2012) also found that generally quantitative methods are preferred in studies in the mathematics education field. In this respect, while quantitative methods are preferred in mathematics education studies, it is found that mainly qualitative methods are used in studies on mathematical thinking. Qualitative studies are conducted in the natural environment of the study in an interpretative and holistic manner and the results of the studies are addressed more thoroughly and in multiple aspects (Creswell, 2003). Therefore, it is thought that allowing for more qualitative studies in mathematics education and supporting these with quantitative studies would be rather beneficial in understanding the thought patterns of the individuals, in order for the studies be conducted more in depths and in multiple aspects. However, the results obtained show that fewer mixed studies are conducted on mathematical thinking. Focusing attention on mixed studies becomes more significant to interpret the data in multiple aspects. Therefore, since qualitative and mixed research methods enable more in depth investigation of the reasons underlying the problems, it can be argued that frequent use of these research methods would bring depth to the studies.

When the sample sizes in the studies are examined, it is seen that generally sample sizes comprising 11 to 30 individuals (18.7%) and 201 to 300 individuals (16.7%) are preferred. It is seen that the rate of determining sample sizes larger than 300 is very low. Çiltaş, Güler and Sözbilir (2012) found that researchers generally preferred sample sizes of 31-100 range in the studies in mathematics education field. It is found that generally middle school students (23.7%) and elementary pre-service mathematics teachers (20.3%) are determined as samples. Put it differently, the researchers mostly determined their samples with regard to secondary education. While the rate for selecting secondary pre-service mathematics teachers as sample is 15.3%, the rate of the studies, which selected high school students, is 13.5%. In addition, it is found that the researchers do not generally prefer mathematics teachers (8.5%) as

samples. These results indicate that researchers generally determined pre-service teachers and secondary or elementary school students as samples. It is observed that they preferred teachers as samples with a very low frequency. Therefore, it is thought that it would be quite important, regarding the diversity and versatility of the studies, for the researchers to determine mathematics teachers as their samples more frequently.

CONCLUSION AND SUGGESTIONS

It is found that the researchers mostly use open-ended questions (26.5%) as data collection tools in their studies. Next, they use scales (22.9%) as data collection tools. Interview forms and observations are among the frequently used data collection tools. On the other hand, it is found that they do not use the achievement tests, rubrics and written documents that frequently. This indicates that the researchers generally prefer qualitative data collection tools in their studies on mathematical thinking. Çiltaş, Güler and Sözbilir (2012) stated that in studies in mathematics education field, generally the surveys and achievement tests are used. In addition, they argued that the researchers use generally one (48%) and two (40%) data collection tools. It is recommended that the researchers use more than one data collection tool to increase the reliability of their findings and to obtain results that are more valid. In this way, the data set of their studies would be richer and more consistent. Thus, it would enable conducting studies with high validity and reliability.

When the data analysis methods used in the studies are examined, it is seen that content analysis is used in 20.2% of the studies, t-test in 14.8% and frequency and percentage in 11.7%. In addition, it is found that in 10.6% of the studies the descriptive analysis is used. These data indicate that the researchers use mostly the qualitative data analysis methods in their studies and the rate of using quantitative analysis methods is low.

It is thought that having knowledge about the methods, sample types, data analysis methods of the studies in mathematics education field, in addition to the topics of the studies, would provide the researchers with guidance in their prospective studies. Thus, it can be asserted that investigating the research tendencies of mathematics education researchers and predicting future tendencies is rather important to review holistically the status of the mathematics education studies in Turkey. It is also thought that the results obtained in this study would be beneficial for taking appropriate decisions in future research.

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