Available online at www.sserr.ro
Social Sciences and Education Research Review

(3) 1 3-30 (2016) ISSN 2392-9683

Endogenous Population Dynamics and Economic Growth with Free Trade between Countries

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Abstract

This paper builds a model to deal with dynamic interdependence between different countries' birth rates, mortality rates, populations, wealth accumulation, and time distributions between working, leisure and children caring. The model shows the role of human capital, technological and preference changes on national differences in birth rates, mortality rates, time distributions, population change, and wealth accumulation. The economic mechanisms of wealth accumulation, production and trade are based the Solow growth model and the Oniki-Uzawa trade model. We use the utility function proposed by Zhang to describe the behavior of households. We model national and gender differences in human capital, propensity to have children, propensity to use leisure time, and children caring efficiency. We describe the dynamics of global economic growth, trade patterns, national differences in wealth, income, birth rates, mortality rates, and populations with differential equations. We simulate the model to show the motion of the system and identify the existence of equilibrium point. We also examine the effects of changes in the propensity to have children, the propensity to save, woman's propensity to use leisure, woman's human capital, and woman's emotional involvement in children caring on the dynamics of the global and national economies.

Keywords: propensity to have children; birth rate; mortality rate; population growth; gender; international trade; time distribution; wealth accumulation
JEL classification: O47, O57

1 INTRODUCTION

This study deals with global dynamic interdependence between economic growth, population growth, and inequalities in income and wealth in a multi-country model with free markets and free trade. It builds a trade model with endogenous wealth, endogenous birth and mortality rates, population dynamics, time distributions between leisure, work and children caring under perfectly competitive markets and free trade. The model is built on the basis of the Solow neoclassical growth model, the Oniki-Uzawa trade model, and some neoclassical growth models with endogenous population. We integrate these approaches by applying the utility function proposed in a unique manner by Zhang to determine saving and consumption.

This study is concerned with global economic dynamics with multiple countries. As far as a country's economy is concerned, this paper is strongly influenced by the Solow model (Solow, 1956; Burmeister and Dobell, 1970; Azariadis, 1993; Barro and Sala-i-Martin, 1995). The model of this study is built within the framework of neoclassical growth theory. We follow the Solow model in modeling economic production and wealth accumulation. Nevertheless, we analyze household behavior by the approach proposed by Zhang (1993). With regards to capital mobility and trade, our model is based on the neoclassical growth trade model. According to Findlay (1984), almost all the trade models developed before the 1960s are static in the sense that the supplies of factors of production are given and do not vary over time. An early trade model with endogenous capital and capital movements is provided by Oniki and Uzawa (1965). They examine trade patterns between two economies in a Heckscher-Ohlin model with fixed savings rates. Deardorff and Hanson (1978) propose a two country trade mode with different saving rates across countries. There are some other growth models with international trade (e.g., Brecher et al., 2002; Nishimura and Shimomura, 2002; Ono and Shibata, 2005). But none of these models with endogenous capital accumulation contain endogenous population changes. It should be also remarked that most of trade models with endogenous capital in the contemporary literature are either limited to two-country or small open economies (for instance, Wong, 1995; Jensen and Wong, 1998; Obstfeld and Rogoff, 1998). It is necessary to deal with trades and growth with an analytical framework with any number of countries as the world does consist of many countries and trades occur among multiple countries. Our model is built for any number of countries.

It is a challenging topic to introduce endogenous population dynamics into economic trade growth model. Even to describe dynamics of a world economy with two national economies, it is almost impossible to avoid higher dimensional nonlinear dynamic analysis. Analysis of higher dimensional differential equations tends to be too complicated with help of computer. This is perhaps a main reason that most of economic growth theories omit dynamics of population, even though interactions between economic growth and population change have been a challenging question in economics even since Malthus published his *An Essay on the Principle of Population* in 1798. This study is to examine dynamic interactions between wealth accumulation and population dynamics with endogenous birth rates and mortality rates in a multi-country neoclassical growth framework. There are many factors, such as changes in gender gap in wages, labor market frictions, and age structure (Galor and Weil, 1999; Adsera, 2005; Hock and Weil, 2012). Bosi and Seegmuller (2012) study heterogeneity of households in terms of capital endowments, mortality, and costs per surviving child. Varvarigos and Zakaria (2013) examine interactions between fertility choice and expenditures on health in the traditional overlapping-generations framework. Another determinant of population change is mortality rate (Robinson and Srinivasan, 1997; Lancia and Prarolo, 2012). This study is influenced by these studies.

Our study is strongly influenced by the literature of the neoclassical growth theory and the literature of population growth and economic development. A unique contribution of this paper is to model population growth of multiple countries in the framework of the Solow growth model with endogenous wealth accumulation and gender time distribution between work, leisure and children caring. The paper analyzes the link between wealth growth, economic growth, gender division of labor, and population growth. The physical capital accumulation is built on the Solow growth model. The birth rate and mortality rate dynamics are influenced by the Haavelmo population model and the Barro-Becker fertility choice model. We synthesize these dynamic mechanisms in a compact framework, applying an alternative utility function proposed by Zhang (1993). The model is actually a synthesis of Zhang's two models. Zhang (1992) develops a model of global growth model with capital accumulation and international trade. Zhang (2015) develops a growth model with endogenous population with a homogenous household. The paper is organized as follows. Section 2 introduces the basic model with wealth accumulation and population dynamics. Section 3 simulates the model. Section 4 carries out comparative dynamic analysis with regard to some parameters. Section 5 concludes the study.

2 THE BASIC MODEL

The model is a synthesis of the two models by Zhang (1992, 2015). The production aspects are based on the Solow growth model (Solow, 1956). The world consists of J countries. Countries are indexed by j, $j = 1, \ldots J$. All the countries produce the internationally homogenous capital good. Each national economy has one sector, producing a single commodity for consumption and investment. In country j capital depreciates at a constant exponential rate δ_{kj} which is independent of the manner of use. This assumption follows the traditional Oniki-Uzawa trade model. Ikeda and Ono (1992) show that most of trade models with endogenous capital are structured like Oniki-Uzawa trade model and its various extensions with one capital good. Like Zhang (1992), we also follow this traditional analytical framework. Households own assets of the economy and distribute their incomes to consume and save. The production sectors use capital and labor. Exchanges take place in perfectly competitive markets. Factor markets work well; factors are inelastically supplied and the available factors are fully utilized at every moment. Saving is undertaken only by households, which implies that all earnings of firms are distributed in the form of payments to factors of production. We omit the possibility of hoarding of output in the form of non-productive inventories held by households. We require savings and investment to be equal at any point in time. Let prices be measured in terms of the capital good and the price of the capital good be unit. We denote wage and interest rates by $w_j(t)$ and $r_j(t)$, respectively, in the j th country. In the free trade system, the interest rate is identical throughout the world economy, i.e., $r(t) = r_j(t)$. Households own assets of the economy and distribute their incomes to consumption, child bearing, and wealth accumulation. The population of each gender is homogeneous. We assume that each family consists of husband, wife and children. All the families are identical. We use subscripts $q = 1$ and $q = 2$ to stand for man and woman respectively. We use $N_j(t)$ to stand for the population of each gender in country j. Country j has the population, $2N_j(t)$. We use (j, q) to index a person of gender q in country j. Let $T_{jq}(t)$ and $\overline{T}_{jq}(t)$ stand for work time and time spent on taking care of children of (j, q) and $\overline{N}_j(t)$ for country j's flow of labor services used in time t for production. We have $\overline{N}_j(t)$ as follows

$$\bar{N}_j(t) = \sum_{j=1}^{J} \left[h_{j1} T_{j1}(t) + h_{j2} T_{j2}(t)\right] N_j(t), \quad j = 1, \ldots, J, \tag{1}$$

where h_{jq} is the level of human capital of person (j, q).

The production sector in country j

The production sector uses capital and labor as inputs. Let $K_j(t)$ stand for the capital stock employed by country j at time t. We take on the following form of the production function

$$F_j(t) = A_j K_j^{\alpha_j}(t) \bar{N}^{\beta_j}(t), \quad \alpha_j, \beta_j > 0, \quad \alpha_j + \beta_j = 1, \tag{2}$$

where $F_j(t)$ is the output level, and A_j, α_j, and β_j are parameters. Markets are competitive; thus labor and capital earn their marginal products. The marginal conditions are

$$r(t) + \delta_{kj} = \frac{\alpha_j F_j(t)}{K_j(t)}, \quad w_j(t) = \frac{\beta_j F_j(t)}{\bar{N}_j(t)}, \quad w_{jq}(t) = h_{jq} w_j(t). \tag{3}$$

Consumer behaviors

We use an alternative approach to household proposed by Zhang (1993). Households decide time distribution, consumption level of commodity, number of children, and amount of saving. To describe behavior of consumers, we denote per family wealth by $\bar{k}_j(t)$. Per family current income from the interest payment and the wage payments is

$$y_j(t) = r(t) \bar{k}_j(t) + \left[h_{j1} T_{j1}(t) + h_{j2} T_{j2}(t)\right] w_j(t).$$

We call $y_j(t)$ the current income in the sense that it comes from consumers' payment for efforts and consumers' current earnings from ownership of wealth. The total value of wealth that a family can sell to purchase goods and to save is equal to $\bar{k}_j(t)$. Here, we assume that selling and buying wealth can be conducted instantaneously without any transaction cost. The disposable income per family is given by

$$\hat{y}_j(t) = y_j(t) + \bar{k}_j(t). \tag{4}$$

Let $n_j(t)$ and $p_{bj}(t)$ stand for the birth rate and the cost of birth. According to Soares (2005) parents' utility depends not only on their surviving offspring, but also on length of each surviving child's lifespan. Following Zhang (2015), we assume that children will have the same level of wealth as that of the parent. In addition to the time spent on children, the cost of the parent is given by

$$p_{bj}(t) = n_j(t)\bar{k}_j(t). \tag{5}$$

Here, we neglect other costs such as purchases of goods and services. We consider the following relation between fertility rate and the parent's time on raising children

$$\bar{T}_{jq}(t) = \theta_{jq}\, n_j(t), \;\; \theta_{jq} \ge 0. \tag{6}$$

The specified function form implies that if the parents want more children, they spend more time on child caring. This linear formation is a strict requirement as child caring tends to exhibit increasing return to scale. For instance, the time per child tends to fall as the family has more children. Clothes and housing are shared among family members. In this stage of modelling we assume constant return to scale because this assumption makes the analysis mathematically tractable.

The household distributes the total available budget between saving, $s_j(t)$, consumption of goods, $c_j(t)$, and bearing children, $p_{bj}(t)$. The budget constraint is

$$p_j(t)c_j(t) + s_j(t) + \bar{k}_j(t)n_j(t) = \hat{y}_j(t). \tag{7}$$

We consider that except work and child caring, parents also have their leisure. We denote the leisure time of person (j, q) by $\widetilde{T}_{jq}(t)$. Each person is faced with the following time constraint

$$T_{jq}(t) + \overline{T}_{jq}(t) + \widetilde{T}_{jq}(t) = T_0 , \tag{8}$$

where T_0 is the total available time for leisure, work and children caring. Substituting (8) into (7) yields

$$p(t)c_j(t) + s_j(t) + \bar{k}_j(t)n_j(t) + \overline{T}_{j1}(t)w_{j1}(t) + \overline{T}_{j2}(t)w_{j2}(t) + \widetilde{T}_{j1}(t)w_{j1}(t) + \widetilde{T}_{j2}(t)w_{j2}(t) = \bar{y}_j(t \tag{9}$$

where

$$\bar{y}_j(t) \equiv (1 + r(t))\bar{k}_j(t) + (w_{j1}(t) + w_{j2}(t))T_0 .$$

The right-hand side is the "potential" income that the family can obtain by spending all the available time on work. The left-hand side is the sum of the consumption cost, the saving, the opportunity cost of bearing children, and opportunity cost of leisure. Insert (6) in (9)

$$c_j(t) + s_j(t) + \widetilde{w}_j(t)n_j(t) + \widetilde{T}_{j1}(t)w_{j1}(t) + \widetilde{T}_{j2}(t)w_{j2}(t) = \bar{y}_j(t), \tag{10}$$

where

$$\widetilde{w}_j(t) \equiv \bar{k}_j(t) + h_j\, w_j(t), \quad h_j \equiv \theta_{j1}\, h_{j1} + \theta_{j2}\, h_{j2} .$$

The variable $\widetilde{w}_j(t)$ is the opportunity cost of children fostering. We assume that the parents' utility is dependent on the number of children. We assume that the utility is dependent on $c_j(t)$, $s_j(t)$, $\widetilde{T}_{jq}(t)$, and $n_j(t)$ as follows

$$U_j(t) = c_j^{\xi_{j0}}(t)s_j^{\lambda_{j0}}(t)\widetilde{T}_{j1}^{\sigma_{j01}}(t)\widetilde{T}_{j2}^{\sigma_{j02}}(t)n_j^{\upsilon_{j0}}(t),$$

where ξ_{j0} is called the propensity to consume, λ_{j0} the propensity to own wealth, σ_{j0q} the gender $q's$ propensity to use leisure time, and υ_{j0} the propensity to have children. The first-order condition of maximizing $U_j(t)$ subject to (13) yields

$$c_j(t) = \xi_j \bar{y}_j(t), \;\; s_j(t) = \lambda_j \bar{y}_j(t), \;\; \tilde{T}_{jq}(t) = \frac{\sigma_{jq} \bar{y}_j(t)}{w_{jq}(t)}, \;\; n_j(t) = \frac{\upsilon_j \bar{y}_j(t)}{\tilde{w}_j(t)}, \tag{11}$$

where

$$\xi_j \equiv \rho_j \xi_{j0}, \;\; \lambda_j \equiv \rho_j \lambda_{j0}, \;\; \sigma_{jq} \equiv \rho_j \sigma_{jq0}, \;\; \upsilon_j \equiv \rho_j \upsilon_{j0}, \;\; \rho_j \equiv \frac{1}{\xi_{j0} + \lambda_{j0} + \sigma_{j10} + \sigma_{j20} + \upsilon_{j0}}.$$

The birth and mortality rates and population dynamics

According to the definitions, the population change follows

$$\dot{N}_j(t) = \left(n_j(t) - d_j(t)\right) N_j(t), \tag{12}$$

where $n_j(t)$ and $d_j(t)$ are respectively the birth rate and mortality rate. Our modelling of birth and mortality rates is influenced by different ideas in the literature of economic growth and population change (e.g., Haavelmo, 1954; Razin and Ben-Zion, 1975; Stutzer, 1980; Yip and Zhang, 1997; Chu *et al.*, 2012; and Tournemaine and Luangaram, 2012). Following Zhang (2015), we assume that the mortality rate is negatively related to the disposable income in the following way

$$d_j(t) = \frac{\bar{\upsilon}_j N_j^{b_j}(t)}{\bar{y}_j^{a_j}(t)}, \tag{13}$$

where $\bar{\upsilon}_j \geq 0$, $a_j \geq 0$. We call $\bar{\upsilon}_j$ the mortality rate parameter. As in the Haavelmo model, an improvement in living conditions implies that people live longer. The term $N_j^{b_j}(t)$ takes account of possible influences of the population on mortality. For instance, when the population is overpopulated, environment is deteriorated. We may take account of this kind of environmental effects by the term. In this case, it is reasonable require b_j to be positive. It should be noted that the sign of b_j is generally ambiguous in the sense that the population may also have a positive impact on mortality. Insert (10) and (13) in (12)

$$\dot{N}_j(t) = \left(\frac{\upsilon_j \bar{y}_j(t)}{\tilde{w}_j(t)} - \frac{\bar{\upsilon}_j N_j^{b_j}}{\bar{y}_j^{a_j}}\right) N_j(t). \qquad (14)$$

The equation describes the population dynamics.

Wealth dynamics

We now find dynamics of wealth accumulation. According to the definition of $s_j(t)$, the change in the household's wealth is given by

$$\dot{\bar{k}}_j(t) = s_j(t) - \bar{k}_j(t) = \lambda_j \bar{y}_j(t) - \bar{k}_j(t). \qquad (15)$$

The value of physical wealth and capital

The value of global physical capital is equal to the value of global wealth

$$\sum_{j=1}^{J} \bar{k}_j(t) N_j(t) = K(t). \qquad (16)$$

Global capital being fully employed

The assumption that the global capital is fully employed implies

$$\sum_{j=1}^{J} K_j(t) = K(t). \qquad (17)$$

Demand for and supply of goods

As the global output is the sum of the net savings and the depreciations of capital, we have

$$S(t) - K(t) + \sum_{j=1}^{J} \delta_{kj} K_j(t) = \sum_{j=1}^{J} F_j(t),$$

(18)

where $S(t) - K(t) + \sum_j \delta_{kj} K_j(t)$ is the sum of the net saving and depreciation and

$$S(t) = \sum_{j=1}^{J} s_j(t) N_j(t), \quad C(t) = \sum_{j=1}^{J} c_j(t) N_j(t).$$

The trade balances of the economies are given by

$$B_j(t) = \left(\bar{K}_j(t) - K_j(t)\right) r(t). \tag{19}$$

When $B_j(t)$ is positive (negative), we say that country j is in trade surplus (deficit). When $B_j(t)$ is zero, country $j's$ trade is in balance.

We have thus built the dynamic model. It can be seen that the model is a unification of the neoclassical growth theory, the neoclassical growth model with trade, and some ideas in the literature of population dynamics with Zhang's approach to the household behavior. For instance, the Solow model, the neoclassical one-sector growth trade model, and the Haavelmo model can be considered as special cases of our model. Moreover, as our model is based on the some well-known mathematical models and includes some features which no other single theoretical model explains, we should be able to explain some interactions which other formal models fail to explain. We now examine dynamics of the model.

3 THE DYNAMICS AND ITS PROPERTIES

This section examines dynamics of the model. First, we introduce

$$z_1(t) \equiv \frac{r(t) + \delta_{k1}}{w_1(t)}.$$

We show that the dynamics can be expressed by differential equations with $z_1(t)$, $\{\bar{k}_j(t)\}$ and $(N_j(t))$ as the variables. Here, $\{\bar{k}_j(t)\} \equiv \left(\bar{k}_2(t), \ldots, \bar{k}_J(t)\right)$.

Lemma

The dynamics of the economic system is governed by

$$\dot{z}_1(t) = \Omega_1\left(z_1(t), \left\{\bar{k}_j(t)\right\}, \left(N_j(t)\right)\right),$$
$$\dot{\bar{k}}_j(t) = \Omega_j\left(z_1(t), \left\{\bar{k}_j(t)\right\}, \left(N_j(t)\right)\right), \quad j = 2, \ldots, J,$$
$$\dot{N}_j(t) = \Lambda_j\left(z_1(t), \left\{\bar{k}_j(t)\right\}, \left(N_j(t)\right)\right), \quad j = 1, \ldots, J. \tag{20}$$

where $\Omega_j(t)$ and $\Lambda_j(t)$ are functions of $z_1(t)$, $\left(\bar{k}_j(t)\right)$ and $\left(N_j(t)\right)$ defined in the Appendix. Moreover, all the other variables are determined as functions of $z_1(t)$, $\left(\bar{k}_j(t)\right)$ and $\left(N_j(t)\right)$: $r(t)$ and $w_{jq}(t)$ by (A2) → $\bar{k}_1(t)$ by (A13) → $\bar{N}_j(t)$ by (A7) → $\bar{y}_j(t)$ by (A3) → $c_j(t)$, $s_j(t)$, $\widetilde{T}_{jq}(t)$, and $n_j(t)$ by (11) → $\bar{T}_{jq}(t)$ by (6) → $T_{jq}(t)$ by (A6) → $K(t)$ by (A7) → $F_j(t)$ by (2).

The differential equations system (20) has $2J$ variables. As demonstrated in the Appendix, the expressions are complicated. It is difficult to explicitly interpret economic implications of the equations. We simulate the model to illustrate behavior of the system. We specify the parameters as follows

$$\alpha_j = 0.25, \ b_j = 0.5, \ a_j = 0.4, \ \delta_{k1} = \delta_{k2} = 0.05, \ \delta_{k3} = 0.06, \ T_0 = 24,$$

$$\begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix} = \begin{pmatrix} 1.2 \\ 1 \\ 0.8 \end{pmatrix}, \begin{pmatrix} h_{11} \\ h_{21} \\ h_{31} \end{pmatrix} = \begin{pmatrix} 2 \\ 1.5 \\ 1 \end{pmatrix}, \begin{pmatrix} h_{12} \\ h_{22} \\ h_{32} \end{pmatrix} = \begin{pmatrix} 1.5 \\ 1 \\ 0.5 \end{pmatrix}, \begin{pmatrix} \xi_{10} \\ \xi_{20} \\ \xi_{30} \end{pmatrix} = \begin{pmatrix} 0.2 \\ 0.2 \\ 0.2 \end{pmatrix}, \begin{pmatrix} \lambda_{10} \\ \lambda_{20} \\ \lambda_{30} \end{pmatrix} = \begin{pmatrix} 0.6 \\ 0.55 \\ 0.5 \end{pmatrix}, \begin{pmatrix} v_{10} \\ v_{20} \\ v_{30} \end{pmatrix} = \begin{pmatrix} 0.5 \\ 0.6 \\ 0.7 \end{pmatrix},$$

$$\begin{pmatrix} \sigma_{110} \\ \sigma_{210} \\ \sigma_{310} \end{pmatrix} = \begin{pmatrix} 0.1 \\ 0.12 \\ 0.16 \end{pmatrix}, \begin{pmatrix} \sigma_{120} \\ \sigma_{220} \\ \sigma_{320} \end{pmatrix} = \begin{pmatrix} 0.13 \\ 0.14 \\ 0.18 \end{pmatrix}, \begin{pmatrix} \bar{v}_1 \\ \bar{v}_2 \\ \bar{v}_3 \end{pmatrix} = \begin{pmatrix} 0.02 \\ 0.03 \\ 0.035 \end{pmatrix}, \begin{pmatrix} \theta_{11} \\ \theta_{21} \\ \theta_{31} \end{pmatrix} = \begin{pmatrix} 0.1 \\ 0.2 \\ 0.1 \end{pmatrix}, \begin{pmatrix} \theta_{12} \\ \theta_{22} \\ \theta_{32} \end{pmatrix} = \begin{pmatrix} 0.5 \\ 0.3 \\ 0.2 \end{pmatrix}. \tag{21}$$

For the same gender, country 1's human capital is higher than country 2's and country 2's human capital is higher than country 3's. The total productivity factors vary between countries. It should be remarked that although the specified values are not based on empirical observations, the choice does not seem to be unrealistic. In many studies (for instance, Miles and Scott, 2005, Abel *et al*, 2007) the value of α in the Cobb-Douglas production function is approximately 0.3. The relative propensities are listed as follows

$$\begin{pmatrix} v_1 \\ \lambda_1 \\ \xi_1 \\ \sigma_{11} \\ \sigma_{12} \end{pmatrix} = \begin{pmatrix} 0.327 \\ 0.392 \\ 0.131 \\ 0.065 \\ 0.085 \end{pmatrix}, \begin{pmatrix} v_2 \\ \lambda_2 \\ \xi_2 \\ \sigma_{21} \\ \sigma_{22} \end{pmatrix} = \begin{pmatrix} 0.373 \\ 0.342 \\ 0.124 \\ 0.075 \\ 0.087 \end{pmatrix}, \begin{pmatrix} v_3 \\ \lambda_3 \\ \xi_3 \\ \sigma_{31} \\ \sigma_{31} \end{pmatrix} = \begin{pmatrix} 0.402 \\ 0.287 \\ 0.115 \\ 0.092 \\ 0.103 \end{pmatrix}.$$

Country 1's propensity to have children is lower than country 2's and country 2's propensity to have children is lower than country 3's. Both man and woman of country 3 have the highest propensities to stay at home. Country 1 has the highest propensity to save. The father has lower propensity to pursue leisure than the mother. In regard to the preference parameters, what are important in our study are their relative values. To follow the motion of the system, we specify the initial conditions

$$z_1(0) = 0.38,\ \bar{k}_2(0) = 34,\ \bar{k}_3(0) = 10,\ N_1(0) = 91200,\ N_2(0) = 39900,\ N_3(0) = 22900.$$

The simulation result is plotted in Figure 1. The population grows from its low initial condition. As the population rate rises, the mortality rate is also increasing. The labor force is increased and the wage rates are reduced. The falling wage rates reduce the opportunity cost of children fostering, resulting in the rise of birth rate. The rising in birth rate is associated with rising in both man's and woman's time of children fostering. As the income falls, both men and women work longer hours. Their leisure hours are reduced. The national wealth and output are increased in association with rising labor force. Nevertheless, both consumption level and wealth per household are reduced. Some studies confirm that there is a decline of the fertility rate alone the process of economic development (e.g., Kirk, 1996; Ehrlich and Lui 1997; Galor, 2012). From Figure 1 we can see that relations between consumption and birth rates vary for different groups in different stages of economic growth.

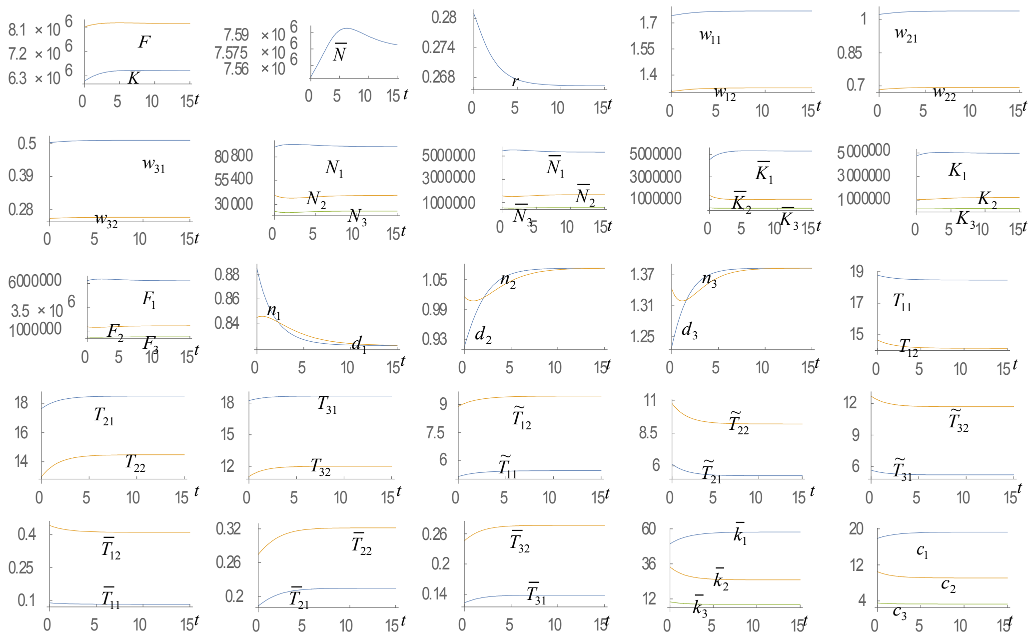

Figure 1. The Motion of the Economic System

It is straightforward to confirm that all the variables become stationary in the long term. This implies the existence of an equilibrium point. The simulation confirms the existence of the following equilibrium point

$K = 6.49\times 10^6$, $\bar{N} = 5.33\times 10^6$, $F = 8.22\times 10^6$, $r = 0.266$, $N_1 = 91667$, $N_2 = 39741$, $N_3 = 22991$, $\bar{N}_1 = 5.33\times 10^6$, $\bar{N}_2 = 1.68\times 10^6$, $\bar{N}_3 = 567067$, $K_1 = 4.97\times 10^6$, $K_2 = 1.23\times 10^6$, $K_3 = 295285$, $F_1 = 6.28\times 10^6$, $F_2 = 1.55\times 10^6$, $F_3 = 385368$, $n_1 = d_1 = 0.82$, $n_2 = d_2 = 1.07$, $n_3 = d_3 = 1.38$, $w_{11} = 1.77$, $w_{12} = 1.33$, $w_{21} = 1.04$, $w_{22} = 0.69$, $w_{31} = 0.51$, $w_{32} = 0.26$, $T_{11} = 18.47$, $T_{21} = 18.53$, $T_{31} = 18.66$, $T_{12} = 14.14$, $T_{22} = 14.48$, $T_{32} = 12.02$, $\tilde{T}_{11} = 5.45$, $\tilde{T}_{21} = 5.25$, $\tilde{T}_{31} = 5.20$, $\tilde{T}_{12} = 9.45$, $\tilde{T}_{22} = 9.19$, $\tilde{T}_{32} = 11.71$, $\bar{T}_{11} = 0.08$, $\bar{T}_{21} = 0.22$, $\bar{T}_{31} = 0.14$, $\bar{T}_{12} = 0.41$, $\bar{T}_{22} = 0.32$, $\bar{T}_{32} = 0.28$, $\bar{k}_1 = 57.86$, $\bar{k}_2 = 25.05$, $\bar{k}_3 = 8.29$, $c_1 = 19.29$, $c_2 = 9.11$, $c_3 = 3.32$.

The countries have different populations. These differences are due to the differences in human capital, the propensities to have children, and other factors. Our dynamic comparative statics will demonstrate how changes in these determining factors will affect the population dynamics. Before effectively conducting dynamic comparative analysis, we guarantee stability of the equilibrium point. We calculate the six eigenvalues

$$-2.97, \ -2.13, \ -1.63, \ -0.68, \ -0.6, \ -0.54.$$

As the eigenvalues are negative, the unique equilibrium is locally stable. Hence, the system always approaches its equilibrium if it is not far from the equilibrium.

4 COMPARATIVE DYNAMIC ANALYSIS IN SOME PARAMETERS BY SIMULATION

We simulated the motion of the national economy under (21). We now examine how the economic system reactions to some exogenous change. As the lemma gives the computational procedure to calibrate the motion of all the variables, it is straightforward to examine effects of change in any parameter on transitory processes as well stationary states of all the variables. Before conducing dynamic comparative analysis, we introduce a variable $\overline{\Delta} x_j(t)$ to stand for the change rate of the variable, $x_j(t)$, in percentage due to changes in the parameter value.

Human capital of country 1's woman being improved

Stotsky (2006: 18) argues that "the neoclassical approach examines the simultaneous interaction of economic development and the reduction of gender inequalities. It sees the process of economic development leading to the reduction of these inequalities and also inequalities hindering economic development." There are many other studies about gender inequalities (e.g., Beneria and Feldman, 1992, Forsythe, et al. 2000). As our model is a general equilibrium model with international trade, endogenous wealth, and and endogenous population, we can deal with complicated relations between different variables. We now examine how the following rise in the human capital of country 1's woman affect the global economic and population dynamics: $h_{12} : 1.5 \Rightarrow 1.7$. The result is plotted in Figure 2. The global labor force, global output, and global capital are enhanced. The rate of interest is reduced. The wage rate of country 1's woman is increased and the wage rates of other groups are slightly increased. Country 1's population and labor force are increased and the other two countries' labor force and population are slightly affected. For country 1, the opportunity cost of child fostering is increased in association with the mother's wage rising. At the same time, the wage rate increased. The net result on the country's birth rate is that it falls. Country's mortality rate falls more than the birth rate till the two rates converge. All the countries produce more and employ more capital. Country 1's level of national wealth is increased and the other two countries' levels of national wealth are slightly affected. Country 1's representative household has more wealth and consume more. The consumption and wealth levels of the other two countries' households are slightly affected. Country 1's man works less hours and woman more hours. The time distributions that country 1's parents spend change correspondingly. The mother from country 1 works more hours and the father works less hours. The father has more leisure time and the mother has less. The parents reduce the hours on children fostering. With regard to the effects on the other two countries' time distributions, each one works longer hours and has less leisure time. The long term effects reduce slightly the two countries' birth and mortality rates.

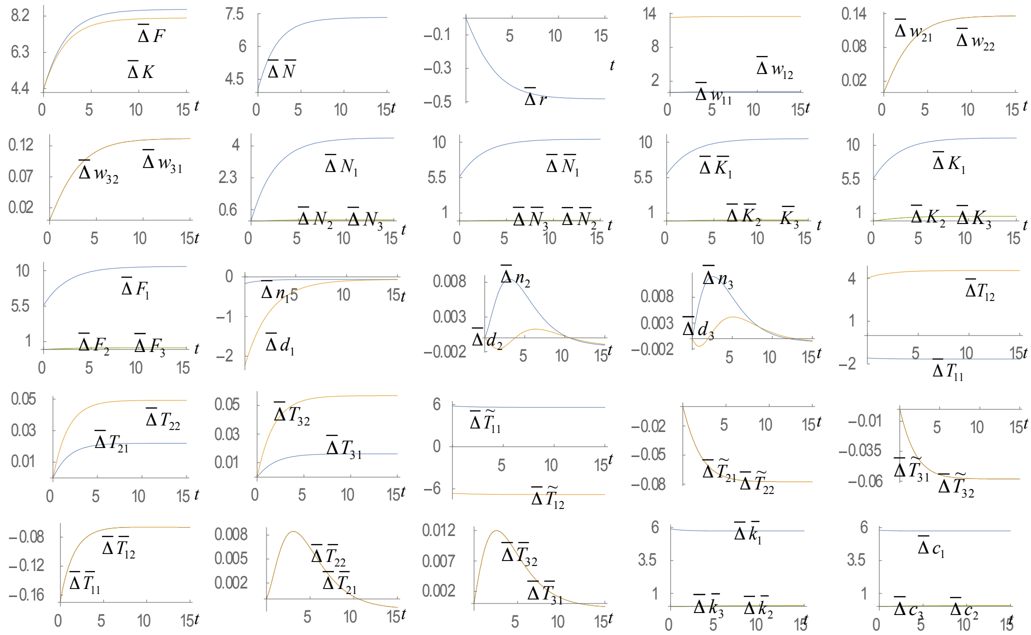

Figure 2. Human Capital of Country 1's Woman Being Improved

Country 1's mother spending more time on per child fostering

We consider what happen to the global economic growth and population dynamics when country 1's mother spends more time on per child fostering in the following way: $\theta_{12}: 0.5 \Rightarrow 0.7$. The result is plotted in Figure 3. Country 1's mother spends more time on children caring and the father slightly changes his time on children caring. Both the father and the mother spend less time on leisure. The mother works less hours and the father works a little more hours. The other two countries' parents slightly increase work hours and reduce leisure time. The global wealth, capital and output are reduced. The rate of interest falls and all the wage rates rise. Country 1's population is reduced and other two countries' populations are slightly increased.

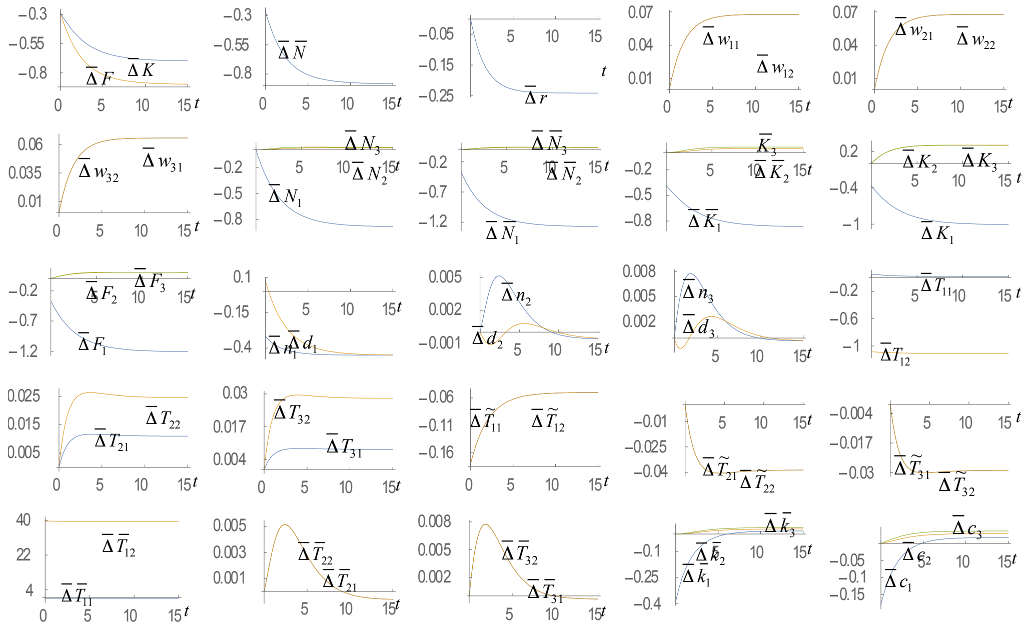

Figure 3. Country 1's Mother Spending More Time on per Child Fostering

A rise in country 1's total factor productivity

We are now concerned with how country 1's total factor productivity affects economic growth and population change. Technological change in the traditional Solow model has positive effects on the long-term economic growth. We now increase country 1's total factor productivity as follows: $A_1 : 1.2 \Rightarrow 1.3$. The simulation results are plotted in Figure 4. As the productivity is enhanced, the global output, wealth and labor force are increased. In the long term all the wage rates are increased. The rate of interest rises initially and falls in the long term. Country 1's population rises. The other two countries' populations are reduced slightly and are affected slightly in the long term. Country 1's birth rate is increased and mortality rate is reduced initially. They are slightly affected in the long term. The other two countries' birth and mortality rates vary initially and are slightly affected in the long term. As shown in Figure 4, country 1's representative household consumes more and holds more wealth.

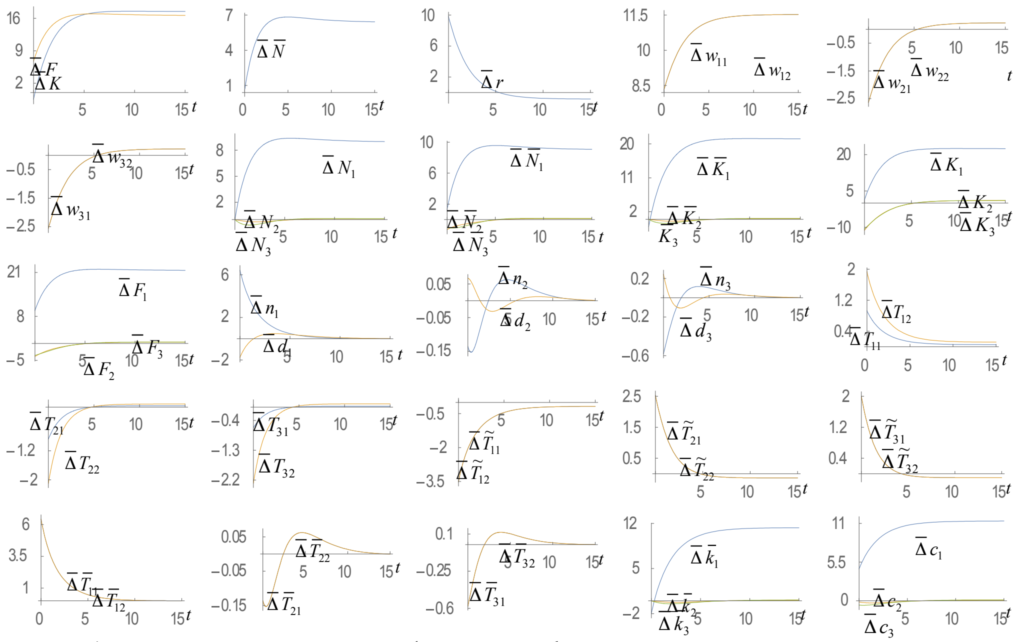

Figure 4. A Rise in Country 1's Total Factor Productivity

A rise in country 1's propensity to have children

Tournemaine and Luangaram (2012: 925) observe that "depending on the country, population growth may contribute, deter, or even have no impact on economic development. This ambiguous result is explained by the fact that the effects of population growth change over time. For example, a higher fertility rate can have a short-term negative effect caused by the cost of expenditures on children whereas it has a long-run positive effect through the larger labor force it generates." As interactions between population change and economic growth are complicated, it is expected that there may be positive, negative or even neutral interdependence between economic growth and population change. We now examine the effects of the following rise in country 1's propensity to have children: $\upsilon_{10}: 0.5 \Rightarrow 0.52$. The simulation results are plotted in Figure 5. As country 1 has more interest in having more children, the country's birth and mortality rates rise. The rise in the mortality rate is due to the reduction in consumption level and the rise in the birth rate is due to the strengthened preference for more children. The net consequence results in the population expansion. We see that the other two countries' mortality and birth rates vary. In the long term the birth and mortality rates in the two countries are increased slightly. The global output, capital and labor force are increased. The rate of interest rises and the wage rates of all the groups fall. Country 1's population is increased and the other two countries' populations are reduced slightly. Country 1 produces more and employs more capital. The other two countries produce less and employ less capital. Country 1 has more wealth and the other two countries have less wealth. Country 1's man and woman work more hours, spend less hours on leisure, and spend more hours on children caring. Country 2's (3's) man and woman work less hours, spend more hours on leisure, and spend less hours on children caring initially and more hours in the long term. The per household consumption and wealth levels are reduced for all the households in the in the global economy.

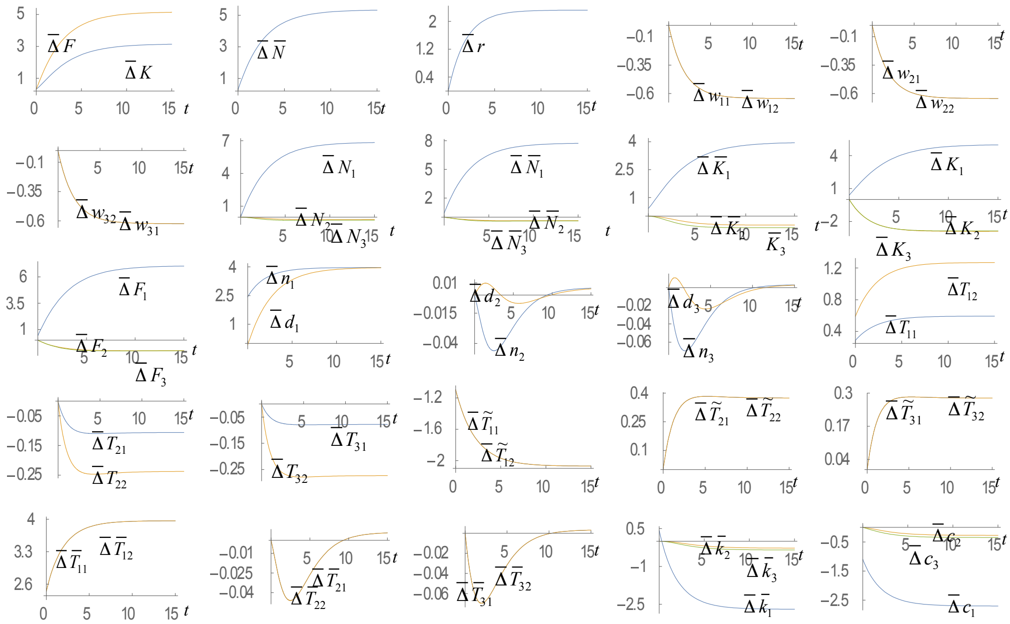

Figure 5. A Rise in Country 1's Propensity to Have Children

Country 1's income having stronger impact on the mortality rate

We now examine what will happen to the economy when country 1's mortality rate is more negatively related to the disposable income in the following way: $a_1 : 0.4 \Rightarrow 0.41$. The result is plotted in Figure 6. Country 1's population and labor force are increased. The other two countries' populations and labor forces are slightly affected. The global labor force, capital and output are increased. The rate of interest falls and the wage rates of all the groups rise. Country 1's output and capital input are increased and the other two countries' output levels and capital inputs are slightly increased. Country 1's mortality rate is reduced and birth rate is slightly affected. The other two countries' birth and mortality rates slightly vary over time. There are also slight changes in the time distributions. The consumption and wealth levels of all the groups are increased.

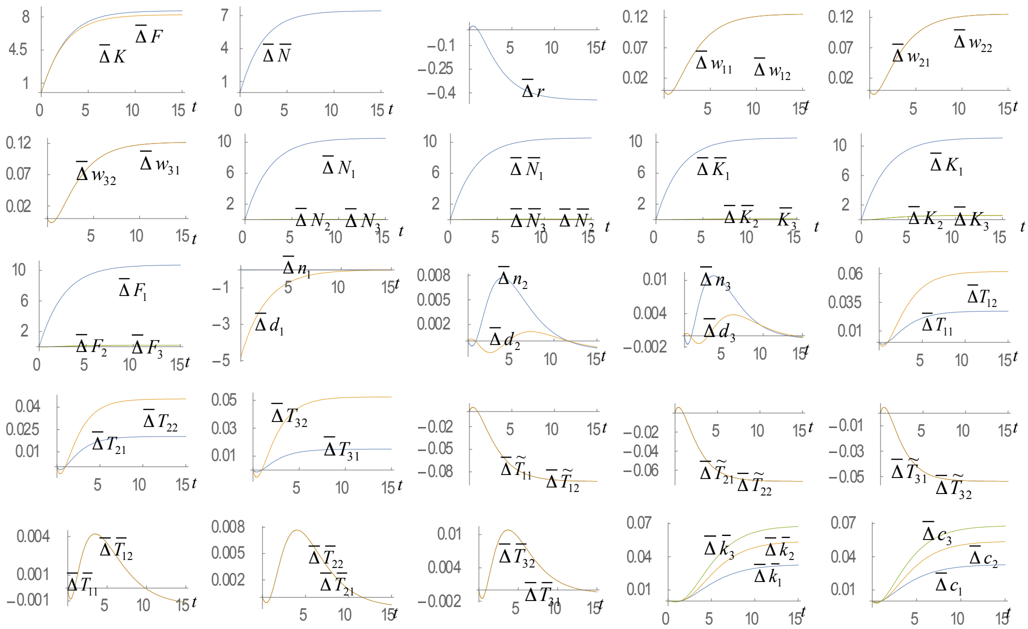

Figure 6. Country 1's Income Having Stronger Impact on the Mortality Rate

A rise in woman's propensity to pursue leisure activities

It is important to examine the economic consequences when woman strengthens her preference for pursuing leisure activities. We now deal with the effects when country 1's woman increases her propensity to pursue leisure as follows: $\sigma_{120}: 0.13 \Rightarrow 0.15$. An immediate consequence of the preference change is that the wife from country 1 spends more time on leisure and the husband has less leisure hours. Country 1's birth and mortality rates are reduced. The husband works more and the wife works less. Both the husband and wife reduce their time of children fostering. The other two countries' households slightly vary their time distributions.

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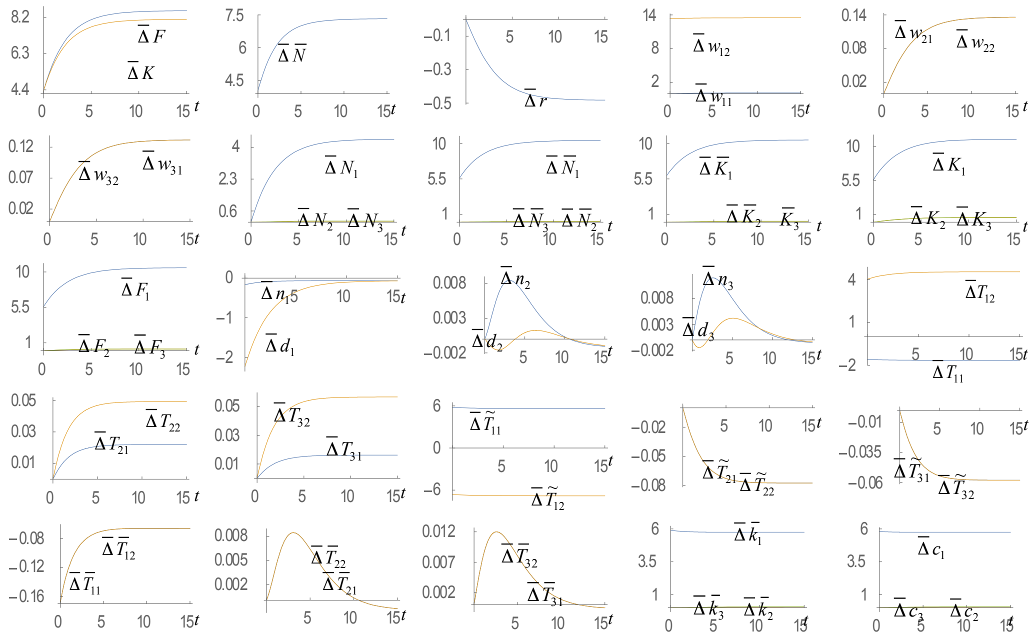

Figure 7. A Rise in Woman's Propensity to Pursue Leisure Activities

Impact of a rise in country 1's propensity to save

According to the Solow model, a rise the propensity to save will increase per capita wealth but reduce per capita consumption level. We will show that the impact in our model is different from the result in the Solow model in the long term. We now allow country 1' propensity to save to be enhanced as follows: $\lambda_{10}: 0.6 \Rightarrow 0.62$. Country 1's per household consumption level falls initially and rises in the long term. The per household wealth in all the countries is augmented. The households in countries 2 and 3 increase their consumption levels. Both man and woman from country 1 work more hours initially and slightly change their work hours in the long term. In the long term the global output, global capital and global labor force are reduced. The rate of interest falls and the wage rates rise. Country 1's population and labor force are reduced. The other two countries' populations and labor forces are increased.

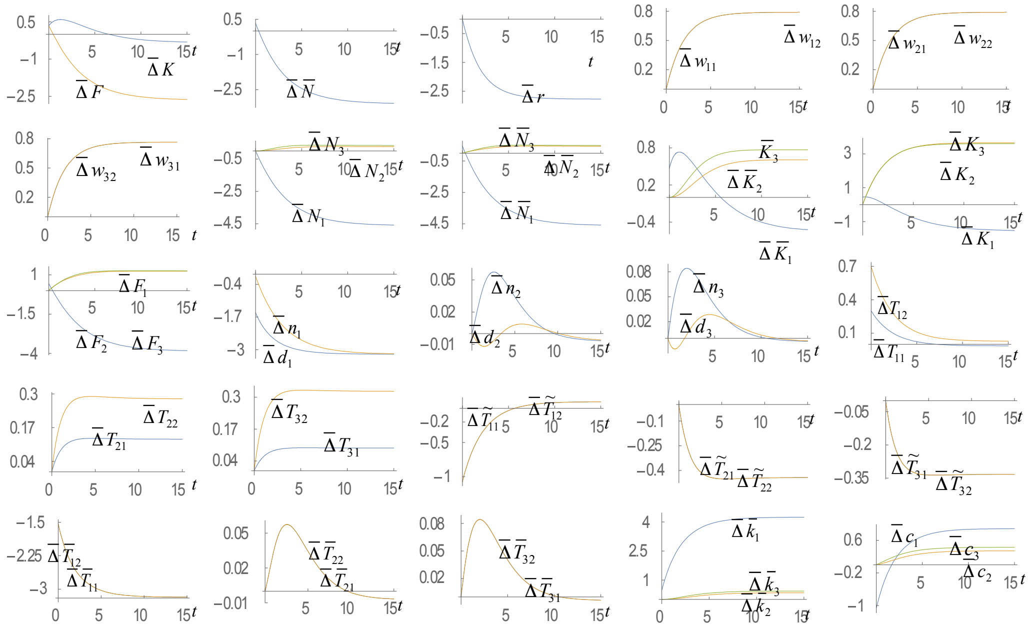

Figure 8. Impact of a Rise in Country 1's Propensity to Save

5 CONCLUDING REMARKS

This paper introduced endogenous population into an Oniki-Uzawa growth model with multiple countries and free trade. The paper is a synthesis of Zhang's models on endogenous population (Zhang, 2015) and on a multi-country growth model with trade (Zhang, 2012). The study dealt with a dynamic interdependence between different countries' birth rates, mortality rates, populations, wealth accumulation, and time distributions between work, leisure and children caring. We emphasized the role of human capital, technological and preference changes on the birth and mortality rates and time distributions. We applied the utility function proposed by Zhang to describe the behavior of households. In our approach the global wealth and income inequality is due to differences in households' preferences and human capital levels as well as the households' initial wealth. We also modelled gender differences in human capital, the propensity to use leisure time, and children caring efficiency. We simulated the model to show the motion of the economic growth and population change and identified the existence of equilibrium point. We examined the effects of changes in the propensity to have children, the propensity to save, woman's propensity to use leisure, woman's human capital, and woman's emotional involvement in children caring. As our model is built on some economic theories, we can generalize and extend our model.

APPENDIX: PROVING THE LEMMA

We now show that the dynamics can be expressed by differential equations system. From (3), we obtain

$$z_j \equiv \frac{r + \delta_{kj}}{w_j} = \frac{\tilde{\alpha}_j \bar{N}_j}{K_j}, \tag{A1}$$

where $\tilde{\alpha}_j \equiv \alpha_j / \beta_j$. From (2) and (3), we have

$$r = \alpha_j A_j \left(\frac{z_j}{\tilde{\alpha}_j}\right)^{\beta_j} - \delta_{kj}, \ \ w_j = \beta_j A_j \left(\frac{\tilde{\alpha}_j}{z_j}\right)^{\alpha_j}, \ \ w_{jq} = h_{jq} w_j. \tag{A2}$$

Hence, we determine r, w_j and w_{jq} as functions of z_j. From (A2) we have

$$r(z_1) = \alpha_1 A_1 \left(\frac{z_1}{\tilde{\alpha}_1}\right)^{\beta_j} - \delta_{kj}, \ \ z_j(z_1) = \tilde{\alpha}_j \left(\frac{r + \delta_{kj}}{\alpha_j A_j}\right)^{1/\beta_j}. \tag{A3}$$

From the definition of $\bar{y}$ and (3) we have

$$\bar{y}_j = (1 + r)\bar{k}_j + h_{j0} w_j, \tag{A4}$$

where $h_{j0} \equiv (h_{j1} + h_{j2})T_0$. By (8) and (11), we have

$$T_{jq} = T_0 - \bar{T}_{jq} - \tilde{T}_{jq} = T_0 - \left(\frac{\theta_{jq} \upsilon_j}{\tilde{w}_j} + \frac{\sigma_{jq}}{h_{jq} w_j}\right)\bar{y}_j. \tag{A5}$$

Insert (A3) in (A4)

$$T_{jq} = \chi_{jq} - \frac{\tilde{r}_{jq} \bar{k}_j + \bar{r}_q}{\tilde{w}_j} - r_{jq} \bar{k}_j, \tag{A6}$$

where

$$\chi_{jq} = \left(1 - \frac{(h_{j1} + h_{j2})\sigma_{jq}}{h_{jq}}\right) T_0, \quad \tilde{r}_{jq} \equiv \theta_{jq} \upsilon_j (1 + r), \quad \bar{r}_{jq} \equiv h_{j0} \theta_{jq} \upsilon_j w_j, \quad r_{jq} \equiv \frac{(1 + r)\sigma_{jq}}{h_{jq} w_j}.$$

Insert (A6) in (1)

$$\bar{N}_j = \left(\chi_j - \frac{\tilde{r}_j \bar{k}_j + \tilde{h}_{3j}}{\tilde{w}_j} - \tilde{r}_{0j} \bar{k}_j\right) N_j, \tag{A7}$$

where

$$\chi_j \equiv h_{j1} \chi_{j1} + h_{j2} \chi_{j2}, \quad \tilde{r}_j \equiv h_{j1} \tilde{r}_{j1} + h_{j2} \tilde{r}_{j2}, \quad \tilde{h}_{3j} \equiv h_{j1} \bar{r}_{j1} + h_{j2} \bar{r}_{j2}, \quad \tilde{r}_{0j} \equiv h_{j1} r_{j1} + h_{j2} r_{j2}.$$

By the definition of z_j, we have

$$K_j = \frac{\tilde{\alpha}_j \bar{N}_j}{z_j}. \tag{A8}$$

From (16) and (17) we have

$$\sum_{j=1}^{J} \bar{k}_j N_j = \sum_{j=1}^{J} K_j. \tag{A9}$$

Insert (A8) in (A9)

$$\sum_{j=1}^{J} \bar{k}_j N_j = \sum_{j=1}^{J} \frac{\tilde{\alpha}_j \bar{N}_j}{z_j}. \tag{A10}$$

Insert (A7) in (A10)

$$\Lambda_0 - \frac{\tilde{r}_1 \bar{k}_1 + \tilde{h}_{31}}{\tilde{w}_1} - \bar{\Lambda}_0 \bar{k}_1 = 0,$$

(A11)

where

$$\Lambda_0\left(z_1, \{\bar{k}_j\}, (N_j)\right) \equiv \chi_1 + \frac{z_1}{\tilde{\alpha}_1 N_1}\left[\sum_{j=2}^{J}\left\{\left(\chi_j - \frac{\tilde{r}_j \bar{k}_j + \tilde{h}_{3j}}{\tilde{w}_j} - \tilde{r}_{0j} \bar{k}_j\right)\frac{\tilde{\alpha}_j}{z_j} - \bar{k}_j\right\} N_j\right],$$

$$\bar{\Lambda}_0(z_1) \equiv \tilde{r}_{01} + \frac{z_1}{\tilde{\alpha}_1}.$$

Insert $\tilde{w}_1 = \bar{k}_1 + h_1 w_1$ in (A11)

$$\bar{k}_1^2 + m_1 \bar{k}_1 + m_2 = 0, \tag{A12}$$

where

$$m_1\left(z_1, \{\bar{k}_j\}, (N_j)\right) \equiv h_1 w_1 + \frac{\tilde{r}_1 - \Lambda_0}{\bar{\Lambda}_0}, \quad m_2\left(z_1, \{\bar{k}_j\}, (N_j)\right) \equiv \frac{\tilde{h}_{31} - h_1 w_1 \Lambda_0}{\bar{\Lambda}_0}.$$

Solve (A12)

$$\bar{k}_1 = \varphi\left(z_1, \{\bar{k}_j\}, (N_j)\right) \equiv \frac{-m_1 \pm \sqrt{m_1^2 - 4m_2}}{2}. \tag{A13}$$

In our simulation the following solution of (A13) has a meaningful solution

$$\bar{k}_1 = \frac{-m_1 + \sqrt{m_1^2 - 4m_2}}{2}.$$

The following procedure shows how to express the variables as functions z_1, $\{\bar{k}_j\}$ and (N_j): r and w_{jq} by (A2) → $\bar{k}_1$ by (A13) → $\bar{N}_j$ by (A7) → $\bar{y}_j$ by (A3) → c_j, s_j, $\tilde{T}_{jq}$, and n_j by (11) → $\bar{T}_{jq}$ by (6) → T_{jq} by (A6) → K by (16) → F_j by (2). From this procedure, (15) and (14), we have

$$\dot{\bar{k}}_1 = \Omega_0\left(z_1, \{\bar{k}_j\}, (N_j)\right) \equiv s_1 - \bar{k}_1, \tag{A14}$$

$$\dot{\bar{k}}_j = \Omega_j\left(z_1, \{\bar{k}_j\}, (N_j)\right) \equiv s_j - \bar{k}_j, \quad j = 2, \ldots, J,$$

$$\dot{N}_j = \Lambda_j\left(z_1, \{\bar{k}_j\}, (N_j)\right) \equiv \left(\frac{\upsilon_j \bar{y}_j}{\tilde{w}_j} - \frac{\bar{\upsilon}_j N_j^{b_j}}{\bar{y}_j^{a_j}}\right) N_j, \quad j = 1, \ldots, J. \tag{A15}$$

Take derivatives of (A13) with respect to t

$$\dot{\bar{k}}_1 = \frac{\partial \varphi}{\partial z_1} \dot{z}_1 + \sum_{j=2}^{J} \frac{\partial \varphi}{\partial \bar{k}_j} \dot{\bar{k}}_j + \sum_{j=1}^{J} \frac{\partial \varphi}{\partial N_j} \dot{N}_j. \tag{A16}$$

Insert (A15) in (A16)

$$\dot{\bar{k}}_1 = \frac{\partial \varphi}{\partial z_1} \dot{z}_1 + \sum_{j=2}^{J} \Omega_j \frac{\partial \varphi}{\partial \bar{k}_j} + \sum_{j=1}^{J} \Lambda_j \frac{\partial \varphi}{\partial N_j}. \tag{A17}$$

Equal the right-hand sides of (A14) and (A17)

$$\dot{z}_1 = \Omega_1\left(z_1, \{\bar{k}_j\}, (N_j)\right) \equiv \left(\Omega_0 - \sum_{j=2}^{J} \Omega_j \frac{\partial \varphi}{\partial \bar{k}_j} - \sum_{j=1}^{J} \Lambda_j \frac{\partial \varphi}{\partial N_j}\right)\left(\frac{\partial \varphi}{\partial z_1}\right)^{-1}.$$

(A18)

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