

# Gauss-Seidel's Theorem for Infinite Systems of Linear Equations (II)

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The purpose of this paper is to extend the classical Gauss-Seidel theorem, known for finite linear systems, to infinite one. First of all we need some technical results [4].

## 1 Vector norms

Let  $x = \begin{pmatrix} x_0 \\ x_1 \\ \vdots \\ x_n \\ \vdots \end{pmatrix}$  be a sequence of real numbers represented in the form of an

infinite column vector, and we denote by  $s$  the real linear space of these sequences.

Let  $p \in [1, +\infty)$  be a real number and define  $l^p = \{x \in s \mid \sum_{i=0}^{\infty} |x_i|^p \text{ is convergent}\}$ .

It is well known that  $l^p$  is a real linear subspace of  $s$  and for every  $x \in l^p$  the formula

$\|x\|_p = \left( \sum_{i=0}^{\infty} |x_i|^p \right)^{1/p}$  defines a norm on  $l^p$ . In this way  $(l^p, \|\cdot\|_p)$  is not only a

normed linear space, but a Banach space, too. For  $p = 1$  and  $p = 2$  we reobtain

the Banach space  $l^1$  and the Hilbert space  $l^2$ , respectively. In  $l^2$  we will consider the

standard scalar product given by the formula  $(x, y) = \sum_{i=0}^{\infty} x_i y_i$  for every  $x, y \in l^2$ .

For  $p, q \in [1, +\infty)$  real numbers from  $p < q$  results  $l^p \subset l^q$ . If  $s_0$  means the linear

subspace of convergent sequences to zero then  $l^p \subset s_0$  for every  $p \in [1, +\infty)$ . We

also consider the linear subspace  $l^\infty = \{x \in s \mid x \text{ is bounded}\}$ . For every  $x \in l^\infty$  the

formula  $\|x\|_\infty = \sup_{i \in \mathbb{N}} \{|x_i|\}$  defines a norm on  $l^\infty$ . In this way  $(l^\infty, \|\cdot\|_\infty)$  is not only a

normed linear space, but a Banach space, too. We have:  $l^1 \subset l^2 \subset s_0 \subset l^\infty \subset s$ . All

these spaces we will call vector spaces, the elements vectors and the above mentioned norms vector norms [1]. For this paragraph see also [4].

## 2 Matrix norms

Let  $A = (a_{ij})_{i,j \in \mathbb{N}}$  be an infinite matrix of real numbers and we denote by  $M$  the real linear space of these infinite matrixes. Let  $M^1 = \left\{ A \in M \mid \sup_{j \in \mathbb{N}} \sum_{i=0}^{\infty} |a_{ij}| \text{ is finite} \right\}$ . Then  $M^1$  is a real linear subspace of  $M$  and for every  $A \in M^1$  the formula  $\|A\|_1 = \sup_{j \in \mathbb{N}} \sum_{i=0}^{\infty} |a_{ij}|$  defines a norm on  $M^1$  called column norm. In this way  $(M^1, \|\cdot\|_1)$  becomes not only a real linear normed space, but a Banach space, too. Let  $p \in (1, +\infty)$  be a real number and define

$$M^p = \left\{ A \in M \mid \sum_{i=0}^{\infty} \left( \sum_{j=0}^{\infty} |a_{ij}|^q \right)^{\frac{p}{q}} \text{ is finite} \right\},$$

where  $q$  is a real number such that  $\frac{1}{p} + \frac{1}{q} = 1$ .

**Theorem 1.** *The space  $M^p$  is a real linear subspace of  $M$  and for every  $A \in M^p$  the formula*

$$\|A\|_p = \left[ \sum_{i=0}^{\infty} \left( \sum_{j=0}^{\infty} |a_{ij}|^q \right)^{\frac{p}{q}} \right]^{\frac{1}{p}}$$

*defines a norm on  $M^p$ . The space  $(M^p, \|\cdot\|_p)$  is a Banach space.*

For  $p = 2$  we obtain  $M^2 = \left\{ A \in M \mid \sum_{i,j=0}^{\infty} a_{ij}^2 \text{ is finite} \right\}$ . If we take on  $M^2$  the scalar product given by the formula  $(A, B) = \sum_{i,j=0}^{\infty} a_{ij}b_{ij}$ , where  $A = (a_{ij})_{i,j \in \mathbb{N}}$  and  $B = (b_{ij})_{i,j \in \mathbb{N}}$ , then  $(M^2, (\cdot, \cdot))$  will be a Hilbert space.

Let  $M^\infty = \left\{ A \in M \mid \sup_{i \in \mathbb{N}} \sum_{j=0}^{\infty} |a_{ij}| \text{ is finite} \right\}$ . Then  $M^\infty$  is a real linear subspace of  $M$  and for every  $A \in M^\infty$  the formula  $\|A\|_\infty = \sup_{i \in \mathbb{N}} \sum_{j=0}^{\infty} |a_{ij}|$  defines a norm on  $M^\infty$ , called row norm. In this way  $(M^\infty, \|\cdot\|_\infty)$  becomes not only a normed linear space, but a Banach space, too.

**Corollary 1.** *If for the matrix  $A = (a_{ij})_{i,j \in \mathbb{N}}$  we have  $a_{ij} = 0$  for  $i > n$  and  $j > n$ ,  $n \in \mathbb{N}$ , then from theorem 1 we reobtain the results in the finite dimensional space  $\mathbb{R}^n$  [3].*

All these spaces we will call matrix spaces and the above mentioned norms matrix norms. For this paragraph see also [4].

### 3 The compatibility of the vector and matrix norms

Let  $x \in s$  be a sequence of real numbers, and  $A = (a_{ij})_{i,j \in \mathbb{N}} \in M$  an infinite matrix of real numbers.

**Definition 1.** *We will define the product  $A \cdot x$  if for every  $i \in \mathbb{N}$  the series  $\sum_{j=0}^{\infty} a_{ij}x_j$  are convergent. In this case the result vector  $y = A \cdot x$  is a column vector with*

$$\text{components } y = \begin{pmatrix} \sum_{j=0}^{\infty} a_{0j}x_j \\ \sum_{j=0}^{\infty} a_{1j}x_j \\ \vdots \\ \sum_{j=0}^{\infty} a_{ij}x_j \\ \vdots \end{pmatrix}$$

**Theorem 2.** *For every  $p \in [1, +\infty] = [1, +\infty) \cup \{+\infty\}$  the vector norm  $\|\cdot\|_p$  defined on  $l^p$  is compatible with the matrix norm  $\|\cdot\|_p$  defined on  $M^p$ , i.e.  $\|Ax\|_p \leq \|A\|_p \cdot \|x\|_p$  for every  $x \in l^p$  and every  $A \in M^p$ .*

**Corollary 2.** *If for the matrix  $A = (a_{ij})_{i,j \in \mathbb{N}}$  we have  $a_{ij} = 0$  for  $i > n$  and  $j > n$ ,  $n \in \mathbb{N}$ , then from theorem 2 we reobtain the results in the finite dimensional space  $\mathbb{R}^n$  [3].*

For this paragraph see also [4].

### 4 The matrix norm subordinate to a given vector norm

For every  $p \in [1, +\infty]$  and for every  $x \in l^p$  and  $A \in M^p$  we have  $\|Ax\|_p \leq \|A\|_p \cdot \|x\|_p$  according to theorem 2. If  $x \neq \theta_{l^p}$  (the null element of the vector space  $l^p$ )

then  $\frac{\|Ax\|_p}{\|x\|_p} \leq \|A\|_p$  and we can define  $\sup \left\{ \frac{\|Ax\|_p}{\|x\|_p} \mid x \in l^p \setminus \{\theta_{l^p}\} \right\}$ . It is known that this formula defines a matrix norm on  $M^p$ , which we call the matrix norm subordinate to the vector norm  $\|\cdot\|_p$  defined on  $l^p$  and we denote by  $\|A\|_p^* = \sup \left\{ \frac{\|Ax\|_p}{\|x\|_p} \mid x \in l^p \setminus \{\theta_{l^p}\} \right\}$ . It is immediately that  $\|A\|_p^* \leq \|A\|_p$  for every  $A \in M^p$ .

**Theorem 3.** For  $p \in \{1, +\infty\}$  we have  $\|A\|_p^* = \|A\|_p$ .

**Corollary 3.** If for the matrix  $A = (a_{ij})_{i,j \in \mathbb{N}}$  we have  $a_{ij} = 0$  for  $i > n$  and  $j > n$ ,  $n \in \mathbb{N}$ , then from theorem 3 we reobtain the results in the finite dimensional space  $\mathbb{R}^n$  [3].

We mention that for the author is unknown how can we calculate for  $p \in (1, +\infty)$  the matrix norm subordinate to the vector norm  $\|\cdot\|_p$  defined on  $l^p$ . For this paragraph see also [4].

The above presented vector and matrix spaces we used to extend the Jacobi's and Gauss-Seidel's methods, known like iterative numerical methods, from finite linear systems to infinite one [5], [6]. In this way we can study the linear stationary processes with infinite but countable number of parameters.

## 5 Gauss-Seidel's iterative method for infinite systems of linear equations

First let us remember the well known Banach fixed point theorem for Banach spaces:

**Theorem 4.** (Banach) Let  $(X, \|\cdot\|_X)$  be a Banach space, and  $\Phi$  a contraction (i.e. there exists a constant  $\alpha \in (0, 1)$  such that  $\|\Phi(x) - \Phi(y)\|_X \leq \alpha \cdot \|x - y\|_X$  for every  $x, y \in X$ ). Then for every  $x^0 \in X$  the sequence  $(x^k)_{k \in \mathbb{N}}$ , generated by the recursion formula  $x^{k+1} = \Phi(x^k)$ , is convergent and has the limit point  $x^* \in X$ , which is the unique fixed point of the function  $\Phi$  in  $X$ .

Let us consider the infinite system of linear equations  $Ax = b$ , where  $A \in M$  and  $x, b \in s$ .

**Definition 2.** For a given  $A \in M$  and  $b \in s$  we will say that  $x^* \in s$  is a solution of the infinite system of linear equations  $Ax = b$  if we have  $Ax^* = b$ .

This means, that all the series  $\sum_{j=0}^{\infty} a_{ij}x_j^*$  are convergent and we have  $\sum_{j=0}^{\infty} a_{ij}x_j^* = b_i$  for every  $i \in \mathbb{N}$ .

Let us suppose, that  $a_{ii} \neq 0$  for every  $i \in \mathbb{N}$ . Then the equation  $\sum_{j=0}^{\infty} a_{ij}x_j = b_i$  is equivalent with the equation

$$x_i = \frac{b_i - \sum_{\substack{j=0 \\ j \neq i}}^{\infty} a_{ij}x_j}{a_{ii}}, \quad \text{i.e.}$$

$$x_i = - \sum_{\substack{j=0 \\ j \neq i}}^{\infty} \frac{a_{ij}}{a_{ii}} x_j + \frac{b_i}{a_{ii}}.$$

So the initial system of linear equations  $Ax = b$  is equivalent with the following iterative system of linear equations:  $x = B \cdot x + c$ , where

$$B = \begin{pmatrix} 0 & -\frac{a_{01}}{a_{00}} & \dots & -\frac{a_{0n}}{a_{00}} & \dots \\ -\frac{a_{10}}{a_{11}} & 0 & \dots & -\frac{a_{1n}}{a_{11}} & \dots \\ \vdots & \vdots & & \vdots & \\ -\frac{a_{n0}}{a_{nn}} & -\frac{a_{n1}}{a_{nn}} & \dots & 0 & \dots \\ \vdots & \vdots & & \vdots & \end{pmatrix} \quad \text{and} \quad c = \begin{pmatrix} \frac{b_0}{a_{00}} \\ \frac{b_1}{a_{11}} \\ \vdots \\ \frac{b_n}{a_{nn}} \\ \vdots \end{pmatrix}.$$

Let us choose  $x^0 \in s$  and we generate the sequence  $(x^k)_{k \in \mathbb{N}} \subset s$  by the following iterative formula:

$$\left\{ \begin{array}{l} x_0^{k+1} = - \sum_{j=1}^{\infty} \frac{a_{0j}}{a_{00}} x_j^k + \frac{b_0}{a_{00}} \\ x_1^{k+1} = - \frac{a_{10}}{a_{11}} x_0^{k+1} - \sum_{j=2}^{\infty} \frac{a_{1j}}{a_{11}} x_j^k + \frac{b_1}{a_{11}} \\ x_2^{k+1} = - \frac{a_{20}}{a_{22}} x_0^{k+1} - \frac{a_{21}}{a_{22}} x_1^{k+1} - \sum_{j=3}^{\infty} \frac{a_{2j}}{a_{22}} x_j^k + \frac{b_2}{a_{22}} \\ \vdots \\ x_i^{k+1} = - \sum_{j=0}^{i-1} \frac{a_{ij}}{a_{ii}} x_j^{k+1} - \sum_{j=i+1}^{\infty} \frac{a_{ij}}{a_{ii}} x_j^k + \frac{b_i}{a_{ii}} \\ \vdots \end{array} \right.$$

Consequently from the vector  $x^k$  we generate the vector  $x^{k+1}$  by the recursion formula  $x^{k+1} = B_{GS}x^k + c$ . Now we consider the following definition:

**Definition 3.** The matrix  $A = (a_{ij})_{i,j \in \mathbb{N}}$  is  $l^\infty$  diagonal dominant if there exists  $\lambda \in (0, 1)$  such that for every  $i \in \mathbb{N}$  we have

$$\lambda \cdot |a_{ii}| > \sum_{\substack{j=0 \\ j \neq i}}^{\infty} |a_{ij}|.$$

It is immediately that  $A$  is  $l^\infty$  diagonal dominant if and only if

$$\sup_{i \in \mathbb{N}} \sum_{\substack{j=0 \\ j \neq i}}^{\infty} \left| \frac{a_{ij}}{a_{ii}} \right| < 1.$$

**Theorem 5.** If  $A$  is  $l^\infty$  diagonal dominant then the iterative sequence  $(x^k)_{k \in \mathbb{N}}$  is convergent in  $l^\infty$  for every  $x^0 \in l^\infty$ . The limit point  $x^* \in l^\infty$  is the unique solution of the linear system  $Ax = b$ .

For this result see also [6].

Here we present another proof for theorem 5.

*Proof.* Let us denote by  $\lambda = \sup_{i \in \mathbb{N}} \sum_{\substack{j=0 \\ j \neq i}}^{\infty} \left| \frac{a_{ij}}{a_{ii}} \right| < 1$ . We prove by mathematical induction method that  $|y_k| \leq \lambda \cdot \|x\|_\infty$  for every  $k \in \mathbb{N}$ , where  $y = B_{GS} \cdot x$ . Indeed,

$$\begin{aligned} |y_0| &= \left| - \sum_{j=1}^{\infty} \frac{a_{0j}}{a_{00}} \cdot x_j \right| \leq \sum_{j=1}^{\infty} \left| \frac{a_{0j}}{a_{00}} \right| \cdot |x_j| \leq \\ &\leq \sum_{j=1}^{\infty} \left| \frac{a_{0j}}{a_{00}} \right| \cdot \|x\|_\infty = \left( \sum_{j=1}^{\infty} \left| \frac{a_{0j}}{a_{00}} \right| \right) \cdot \|x\|_\infty \leq \lambda \cdot \|x\|_\infty. \end{aligned}$$

We suppose that  $|y_j| \leq \lambda \|x\|_\infty$  for every  $j = \overline{0, k-1}$  and we prove that  $|y_k| \leq$

$\lambda \cdot \|x\|_\infty$ . Indeed,

$$\begin{aligned}
|y_k| &= \left| -\sum_{j=0}^{k-1} \frac{a_{kj}}{a_{kk}} \cdot y_j - \sum_{j=k+1}^{\infty} \frac{a_{kj}}{a_{kk}} \cdot x_j \right| \leq \\
&\leq \sum_{j=0}^{k-1} \left| \frac{a_{kj}}{a_{kk}} \right| \cdot |y_j| + \sum_{j=k+1}^{\infty} \left| \frac{a_{kj}}{a_{kk}} \right| \cdot |x_j| \leq \\
&\leq \sum_{j=0}^{k-1} \left| \frac{a_{kj}}{a_{kk}} \right| \cdot \lambda \cdot \|x\|_\infty + \sum_{j=k+1}^{\infty} \left| \frac{a_{kj}}{a_{kk}} \right| \cdot \|x\|_\infty = \\
&= \left( \sum_{j=0}^{k-1} \left| \frac{a_{kj}}{a_{kk}} \right| \cdot \lambda + \sum_{j=k+1}^{\infty} \left| \frac{a_{kj}}{a_{kk}} \right| \right) \cdot \|x\|_\infty \leq \\
&\leq \left( \sum_{j=0}^{k-1} \left| \frac{a_{kj}}{a_{kk}} \right| + \sum_{j=k+1}^{\infty} \left| \frac{a_{kj}}{a_{kk}} \right| \right) \cdot \|x\|_\infty \leq \lambda \cdot \|x\|_\infty,
\end{aligned}$$

because  $\sum_{\substack{j=0 \\ j \neq k}}^{\infty} \left| \frac{a_{kj}}{a_{kk}} \right| \leq \lambda < 1$ . Since  $|y_k| \leq \lambda \cdot \|x\|_\infty$  for every  $k \in \mathbb{N}$  results that

$$\|y\|_\infty = \sup_{k \in \mathbb{N}} \{|y_k|\} \leq \lambda \cdot \|x\|_\infty.$$

This means that

$$\|B_{GS}\|_\infty = \sup_{x \neq \theta_{l^\infty}} \frac{\|B_{GS}x\|_\infty}{\|x\|_\infty} = \sup_{x \neq \theta_{l^\infty}} \frac{\|y\|_\infty}{\|x\|_\infty} \leq \lambda < 1.$$

Now we can apply the Banach fixed point theorem for the iterative function  $\Phi : l^\infty \rightarrow l^\infty$ ,  $\Phi(x) = B_{GS}x + c$ . Indeed,  $\Phi$  is a contraction, because

$$\|\Phi(x) - \Phi(y)\|_\infty = \|(B_{GS}x + c) - (B_{GS}y + c)\|_\infty = \|B_{GS}(x - y)\|_\infty \leq \|B_{GS}\|_\infty \cdot \|x - y\|_\infty.$$

This means that the sequence  $(x^k)_{k \in \mathbb{N}}$  is convergent in  $l^\infty$  for every  $x^0 \in l^\infty$  and its limit point  $x^* \in l^\infty$  is the unique fixed point of  $\Phi$  in  $l^\infty$ , i.e.  $\Phi(x^*) = x^*$ . So  $B_{GS}x^* + c = x^*$ , which is equivalent with  $Ax^* = b$ . □

**Corollary 4.** *If for the matrix  $A = (a_{ij})_{i,j \in \mathbb{N}}$  we have  $a_{ij} = 0$  when  $i > n$ ,  $j > n$ , and  $b_i = 0$  for  $i > n$ ,  $n \in \mathbb{N}$ , then we reobtain the linear system with finite number of equations and finite number of unknowns. In this way from theorem 5 we obtain the classical Gauss-Seidel's iterative numerical method to solve finite systems of linear equations [2].*

In the next we consider the following definition:

**Definition 4.** The matrix  $A = (a_{ij})_{i,j \in \mathbb{N}}$  is  $l^1$  diagonal dominant if there exists  $\lambda \in (0, \frac{1}{2})$  such that for every  $j \in \mathbb{N}$  we have  $\lambda \cdot |a_{jj}| > \sum_{\substack{i=0 \\ i \neq j}}^{\infty} |a_{ij}|$ .

It is immediately that  $A$  is  $l^1$  diagonal dominant if and only if  $\sup_{j \in \mathbb{N}} \sum_{\substack{i=0 \\ i \neq j}}^{\infty} \left| \frac{a_{ij}}{a_{jj}} \right| < \frac{1}{2}$ .

**Theorem 6.** If  $A$  is  $l^1$  diagonal dominant then the iterative sequence  $(x^k)_{k \in \mathbb{N}}$  is convergent in  $l^1$  for every  $x^0 \in l^1$ . The limit point  $x^* \in l^1$  is the unique solution of the linear system  $Ax = b$ .

*Proof.* We have:

$$\begin{aligned} \|y\|_1 &= \sum_{i=0}^{\infty} |y_i| = \sum_{i=0}^{\infty} \left| - \sum_{j=0}^{i-1} \frac{a_{ij}}{a_{ii}} \cdot y_j - \sum_{j=i+1}^{\infty} \frac{a_{ij}}{a_{ii}} \cdot x_j \right| \leq \\ &\leq \sum_{i=0}^{\infty} \left( \sum_{j=0}^{i-1} \left| \frac{a_{ij}}{a_{ii}} \right| \cdot |y_j| + \sum_{j=i+1}^{\infty} \left| \frac{a_{ij}}{a_{ii}} \right| \cdot |x_j| \right) \leq \\ &\leq \sum_{j=0}^{\infty} \left( \sum_{i=0}^{j-1} \left| \frac{a_{ij}}{a_{jj}} \right| \cdot |x_j| + \sum_{i=j+1}^{\infty} \left| \frac{a_{ij}}{a_{jj}} \right| \cdot |y_j| \right) \leq \\ &\leq \sum_{j=0}^{\infty} \left[ \left( \sum_{i=0}^{j-1} \left| \frac{a_{ij}}{a_{jj}} \right| \right) \cdot |x_j| + \left( \sum_{i=j+1}^{\infty} \left| \frac{a_{ij}}{a_{jj}} \right| \right) \cdot |y_j| \right] \leq \\ &\leq \sum_{j=0}^{\infty} (\lambda \cdot |x_j| + \lambda \cdot |y_j|) = \lambda \cdot \|x\|_1 + \lambda \cdot \|y\|_1. \end{aligned}$$

Consequently:  $\|y\|_1 \leq \lambda \cdot \|x\|_1 + \lambda \cdot \|y\|_1$ , which is equivalent with:  $\frac{\|y\|_1}{\|x\|_1} \leq \frac{\lambda}{1-\lambda} < 1$ .

This means that:

$$\|B_{GS}\|_1 = \sup_{x \neq \theta_{11}} \frac{\|B_{GS}x\|_1}{\|x\|_1} = \sup_{x \neq \theta_{11}} \frac{\|y\|_1}{\|x\|_1} \leq \frac{\lambda}{1-\lambda} < 1.$$

Now we can apply the Banach fixed point theorem for the iterative function  $\Phi : l^1 \rightarrow l^1$ ,  $\Phi(x) = B_{GS}x + c$ . Indeed,  $\Phi$  is a contraction, because:  $\|\Phi(x) - \Phi(y)\|_1 = \|(B_{GS}x + c) - (B_{GS}y + c)\|_1 = \|B_{GS}(x - y)\|_1 \leq \|B_{GS}\|_1 \cdot \|x - y\|_1$ . This means that the sequence  $(x^k)_{k \in \mathbb{N}}$  is convergent in  $l^1$  for every  $x^0 \in l^1$  and its limit point  $x^* \in l^1$  is the unique fixed point of  $\Phi$  in  $l^1$ , i.e.  $\Phi(x^*) = x^*$ . So  $B_{GS}x^* + c = x^*$ , which is equivalent with  $Ax^* = b$ .  $\square$

**Corollary 5.** If for the matrix  $A = (a_{ij})_{i,j \in \mathbb{N}}$  we have  $a_{ij} = 0$  when  $i > n$ ,  $j > n$ , and  $b_i = 0$  for  $i > n$ ,  $n \in \mathbb{N}$ , then we reobtain the linear system with finite number of

*equations and finite number of unknowns. In this way from theorem 6 we obtain the classical Gauss-Seidel's iterative numerical method to solve finite systems of linear equations [2].*

We mention that for the author is unknown if theorem 6 is true with definition 4 choosing  $\lambda \in [\frac{1}{2}, 1)$ .

Using the above presented theorems we can study the linear stationary processes with infinite but countable number of parameters.

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