A REMARCABLE PROPERTY OF EXCESSIVE FUNCTIONS CONE

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Abstract: In this paper we present remarcable properties of excessive functions with respect to absolute continue resolvent. The cone of above functions forms a H-cone.

Keywords: excessive function, H-cone, absolute continue resolvent

The excessive functions with respect to absolute continue resolvent.

Definiton 1. If $(X,\mathcal{B})$ is a measurable space and $p\mathcal{B}$ the numerical positive measurable functions on E we denote a kernel on $(X,\mathcal{B})$ a map $V: p\mathcal{B} \to p\mathcal{B}$ with properties:

1. $V0 = 0$
2. $V(\sum_n f_n) = \sum_v V(f_n)$.

Definition 2. A family $\mathcal{V} = (V_\alpha)_{\alpha>0}$ of kernels on measurable space (X,β) is called resolvent if the followings hold:

1. $V_\alpha V_\beta = V_\beta V_\alpha, \ \forall\, \alpha,\beta > 0$
2. $V_\alpha = V_\beta + (\beta-\alpha)V_\alpha V_\beta, \forall\, \alpha,\beta > 0, \alpha < \beta.$

The resolvent is called sub-Markovian if for any $\alpha > 0$ we have $\alpha V_\alpha \leq 1$.
The kernel $V = \sup_\alpha V_\alpha f$ is called initially kernel.

Definition 3. A map $s \in p\mathcal{B}$ is called $\mathcal{V}$- excessive if the followings hold:

1. s is $\mathcal{V}$ –supermedian, i.e $\alpha V_{\alpha s} \leq s$ for any $\alpha > 0$
2. $\sup_\alpha \alpha V_\alpha s = s$
3. s is finite $\mathcal{V}$ a.e. (a set $A \subset X$ is $\mathcal{V}$-negligible if there exists $A' \subset \mathcal{B}$ such that $A \subset A'$ and $V_\alpha(A') = 0$, for any $\alpha > 0$).

Proposition 1. We have the following properties:

1. If $s \in \mathcal{S}_\mathcal{V}$ it follows that $\hat{s} = \sup_\alpha V_\alpha s \in \mathcal{S}_\mathcal{V}$, $\hat{s} \leq s$
2. If $s, t \in \mathcal{S}_\mathcal{V}$ it follows that $\widehat{s+t} = \hat{s} + \hat{t}$
3. For any $(s_n)_n \subset \mathcal{S}_\mathcal{V}$, $s_n \uparrow s$ it follows that $s_n \uparrow s$
4. For any $s \in \mathcal{S}_\mathcal{V}$ it follows that $V_\alpha \hat{s} = V_\alpha s$
5. If $s \in \mathcal{S}_\mathcal{V} \Rightarrow \hat{s} = s$ $\mathcal{V}$- a.p.t.
6. If $s \in \mathcal{S}_\mathcal{V} \Rightarrow \hat{\hat{s}} = s$ see (e.g [1] Proposition 1.18)

Theorem 1. We have the following properties:

1. For any $t \in \mathcal{E}_\mathcal{V}$ (the set of excessive functions), $\alpha, \beta > 0$ it follows that $\alpha s + \beta t \in \mathcal{E}_\mathcal{V}$

2. For any $s, t \in \mathcal{E}_{\mathcal{V}}, s \leq t$ $\mathcal{V}$-a. e. it follows that $s \leq t$
3. For any $(s_n)_n \subseteq \mathcal{E}_v$ there exists $\bigwedge_n s_n$ and we have $\bigwedge_n s_n = \widetilde{\inf} s_n$ and $s + \bigwedge_n s_n = \bigwedge_n (s + s_n)$, for any $s \in \mathcal{E}_{\mathcal{V}}$
4. For any $(s_n)_n \subset \mathcal{E}_{\mathcal{V}}$ dominated there exists $\bigvee_n s_n$ and we have $\bigvee_n s_n = R^{\mathcal{V}}(\sup_n s_n)$, where $R^{\mathcal{V}} f = \inf\{s \in \mathcal{S}_{\mathcal{V}} \mid s \geq f\}$ ($s \in \mathcal{S}_{\mathcal{V}}$ if and only if $\alpha V \alpha s \leq s$, for any $\alpha > 0$) (see e.g. [1] Theorem 1.1.9)

If $(s_n)_n$ is an increasing family we have $\bigvee_n s_n = \sup_n s_n$.

5. For any $s, t, u \in \mathcal{E}_{\mathcal{V}}$ such that $s \leq t + u$, there exists $s', s'' \in \mathcal{E}_{\mathcal{V}}$ such that $s' + s''$ and $s' \leq t, s'' \leq u$. (see e.g. [4] Theorem 1.1.9)

Definition 4. A resolvent $\mathcal{V} = (V_\alpha)_{\alpha > 0}$ is absolutely continue (with respect to a finite measure m) if for any $f \in \mathcal{F}$, such that $\int f dm = 0$ it follows that $V_\alpha f = 0$, for any $\alpha > 0$).

Theorem 2. Let $\mathcal{V} = (V_\alpha)_{\alpha > 0}$ an absolutely continue resolvent (with respect to a finite measure m). Then the followings hold:

1. For any increasingly and dominated family $(s_i)_{i \in I} \subset \mathcal{E}_{\mathcal{V}}$ we have $\sup_{i \in I} s_i \in \mathcal{E}_{\mathcal{V}}$ and there exists an increasing family $(s_{i_n})_{i_n \in I}$, such that $\sup_n s_{i_n} = \sup_{i \in I} s_i = \bigvee_{i \in I} s_i$.
2. For any family $(s_i)_{i \in I} \in \mathcal{E}_{\mathcal{V}}$ there exists a subsequence $(i_n)_n \subset I$, such that $\bigwedge_{i \in I} s_i = \widetilde{\inf} s_{i_n}$ and $s + \bigwedge_{i \in I} s_i = \bigwedge_{i \in I} (s + s_i)$, for any $s \in \mathcal{E}_{\mathcal{V}}$ (see e.g [4] Theorem 1.1.10)

Definition 4. An ordered convex cone S is called H-cone if the following axioms are satisfied:

1. For any non-empty family $A \subset S$ there exists $\bigwedge A$ and we have $s + \bigwedge A = \bigwedge(s + A)$, for any $s \in S$;
2. For any increasing and dominated family $A \subset S$ there exists $\bigvee A$ and we have $\bigvee(A = u) = \bigvee A + u$, for any $u \in S$;
3. S satisfies the Riesz decomposition property i.e. for any $s, s_1, s_2 \in S$ such that $s \leq s_1 + s_2$ there exist $t_1, t_2 \in S$ satisfying $s = t_1 + t_2$, $t_1 \leq s_1, t_2 \leq s_2$.

From two theorem above it follows that the cone of excessive functions with respect to absolute continue resolvent forms a H-cone.

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