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Global Business Cycles with Real Shocks in a General Equilibrium Trade Model with Endogenous Human Capital

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Abstract

This paper generalizes the dynamic economic model of heterogeneous households built by Zhang (2015). The model tries to provide insights into the economic mechanisms of how the richest one per cent of the global population own 50% of the global wealth. It explains inequality in a purely competitive economic environment with endogenous wealth and human capital accumulation. The study simulated a case with three countries and each country with three groups of the population. It demonstrated the existence of an equilibrium point at which the rich 1% own more than half of the global wealth. This study generalizes Zhang's model by making all the time-independent parameters as time-dependent parameters. We demonstrate how the system reacts to exogenous periodic perturbations.

Keywords: Business cycles; exogenous shocks; inequality and global growth; trade; endogenous wealth; endogenous human capital;

1 Introduction

There are different ideas and theories to explain well-observed business cycles and economic oscillations. The two contemporary theories of business cycles are the Keynesian economics and the real business-cycle theory. The Keynesian economic business cycle theory considers demand changes as the main sources of business cycles. The real business-cycle theory explains business-cycle fluctuations by real shocks. Different from other influential business-cycle theories, the real business-cycle theory considers business cycles as the efficient response to exogenous changes in the real economic variables. The purpose of this study is to demonstrate existence of business cycles in the dynamic growth model of heterogeneous households with wealth accumulation and human capital accumulation in a multi-country global economy proposed by Zhang (2015). The paper shows how fluctuations in one country causes business cycles not only in the national economy but also in all the countries in the global economy with free trade. It should be noted that modern nonlinear dynamic economic theory shows that business cycles may occur in a dynamic system with or without any exogenous influences (e.g., Zhang, 1991, 2005, 2006; Lorenz, 1993; Flaschel *et al* 1997; Chiarella and Flaschel, 2000; Shone, 2002; Gandolfo, 2005; Puu, 2011; Tian, 2015). Different studies explain economic business cycles from different perspectives. Lucas (1977) demonstrates how some shocks affect all sectors in an economy. In the neoclassical growth model by Chatterjee and Ravikumar (1992), seasonal perturbations to taste and technology cause business cycles. Gabaix (2011) shows that uncorrelated sectoral shocks are determinants of aggregate fluctuations (see also, Giovanni, et al. 2014; Stella, 2015). This study attempts to identify economic fluctuations due to exogenous shocks in preferences and technologies.

It has been reported that the richest 1% of the world population own almost half of the world's wealth. Moreover, it does seem that inequality be enlarged in the near future in tandem with rapid economic globalization. There is a need to know determinants of inequality and dynamics of inequality. This need is emphasized by Forbes (2000) as follows: "careful reassessment of the relationship between these two variables (growth rate and income inequality) needs further theoretical and empirical work evaluating the channels through which inequality, growth, and any other variables are related." Zhang (2015) has recently built a model to insights into the well-reported phenomena that the richest 1% of the world population owns almost half of the world's wealth. The study is primarily concerned with dynamic interdependence between time distribution between work and leisure, wealth and physical capital accumulation, human capital accumulation, and trade patterns in a multi-country neoclassical growth theory framework. The wealth and income distributions are according to the Walrasian general equilibrium model. The Walrasian general equilibrium theory was initially developed by Walras (Walras, 1874). The theory was further developed and refined by many economists (e.g., Arrow and Debreu, 1954; Gale, 1955; Nikaido, 1956, 1968; Debreu, 1959; McKenzie, 1959; Arrow and Hahn, 1971; Arrow, 1974; and Mas-Colell *et al.*, 1995). The theory solves equilibrium of pure economic exchanges with heterogeneous supplies and households. From the perspective of modern economies the theory has a serious shortcoming which is failures of properly including endogenous wealth (and other factors such as environment, resources, human capital and knowledge) irrespective many attempts done by many economists. Walras did not succeed in developing a general equilibrium theory with endogenous saving and capital accumulation (e.g., Impicciatore *et al.*, 2012). Over years many economists attempted to further develop Walras' capital accumulation within Walras' framework (e.g., Morishima, 1964, 1977; Diewert, 1977; Eatwell, 1987; Dana et al. 1989; and Montesano, 2008).

The global growth forces are the physical capital and human capital accumulation. In Zhang's approach the growth mechanism of physical accumulation and production sides are influenced by the Solow growth model. The modelling of international trade are influenced by the dynamic trade models with accumulating capital developed by Oniki and Uzawa and others (e.g., Oniki and Uzawa, 1965; Frenkel and Razin, 1987; Sorger, 2002; and Nishimura and Shimomura, 2002). Nevertheless, most of trade models with endogenous capital are still either limited to two-country or small open economies without taking account of endogenous human capital (for instance, Grossman and Helpman, 1991). Zhang's model is built for any number of national economies. The study is concerned with not only inequalities in income, wealth and economic structures between (any number of) countries, but also differences in human capital between countries. It is significant to examine dynamic interdependence between economic growth and human capital. The main force of economic growth is capital accumulation in the neoclassical growth theory. But the dynamics of this single variable fails to properly explain why countries grow differently (Easterlin, 1981). The dynamics of human capital is considered another key determinant of economic growth (Hanushek and Kimko, 2000; Barro, 2001; Krueger and Lindahl, 2001; Castelló-Climent and Hidalgo-Cabrillana, 2012; and Barro and Lee, 2013; Hanushek *et al.* 2014). Zhang (2015) integrated the approaches in the neoclassical growth theory and the growth theory with human capital to explain the dynamics of global growth and inequalities in income and wealth. This study generalizes Zhang's model by allowing all the time-dependent parameters to be time-dependent. By the generalization we are able to examine effects of exogenous fluctuations in any parameter on the motion of the system. The rest of this paper is organized as follows. Section 2 defines the neoclassical growth model of a multi-country economy with capital accumulation and human capital accumulation. Section 3 shows that the dynamics of the economy can be described by a set of dimensional differential equations. As mathematical analysis of the system is too complicated, we demonstrate some of the dynamic properties by simulation when the economy consists of three national economies. Section 4 carries out comparative dynamic analysis with exogenous shocks in some parameters. Section 5 concludes the study.

2 The basic model

This section is to generalize Zhang's multi-country model by allowing all the time-independent parameters to be time-dependent (Zhang, 2015). The generalization allows us to analyze time-dependent shocks. We assume that all the economy can produce a homogenous tradable commodity (see also Ikeda and Ono, 1992). Most aspects of production sectors in our model are similar to the neoclassical one-sector growth model. Households own assets of the economy and distribute their incomes to consume and save. Production sectors or firms use capital and labor. Exchanges take place in perfectly competitive markets. Production sectors sell their product to households or to other sectors and households sell their labor and assets to production sectors. Factor markets work well; factors are inelastically supplied and the available factors are fully utilized at every moment. Saving is undertaken only by households, which implies that all earnings of firms are distributed in the form of payments to factors of production. We omit the possibility of hoarding of output in the form of non-productive inventories held by households. All savings volunteered by households are absorbed by firms. We require savings and investment to be equal at any point of time. Commodities are traded without any barriers such as transport costs or tariffs. We assume that there is no migration between the countries. Let prices be measured in terms of the commodity and the price of the commodity be unity. We assume that the population of country j can be classified into Q_j groups, indexed by q, according to their preferences, wealth, human capital, and social status. The total number of types of households Q in the world economy is given by

$$Q = \sum_{j=1}^{J} Q_j .$$

A group q in country j is indexed by (j, q). We introduce

$$Q^* \equiv \{(j, q) \mid j = 1, \cdots, J, \ q = 1, \cdots, Q_j\}.$$

Let the population of group q in country j be $N_{jq}(t)$ in time t. Let $T_{jq}(t)$ stand for the work time of a typical worker in group (j, q). The variable $N_j(t)$ represents the total qualified labor force in country j. A worker's labor force is $T_{jq}(t)H_{jq}^{m_{jq}(t)}(t)$, where $m_{jq}(t)$ is a time-dependent parameter measuring utilization efficiency of human capital by group (j, q). The labor input is the work time by the effective human capital. A group's labor input is the group's population by each member the labor force, that is, $T_j(t)H_j^{m_j(t)}(t)N_{jq}(t)$. As the total qualified labor force is the sum of all the groups' labor forces, we have country j 's total labor force $N_j(t)$ as follows

$$N_j(t) = \sum_{q=1}^{Q_j} T_{jq}(t)H_{jq}^{m_{jq}(t)}(t)N_{jq}(t), \qquad q = 1, ..., Q.$$

(1)

where $H_{jq}(t)$ is the level of human capital of group (j, q). We denote wage and interest rates by $w_{jq}(t)$ and $r_j(t)$, respectively, in the j th country. In the free trade system, the interest rate is identical throughout the world economy, i.e., $r(t) = r_j(t)$.

Production sectors

The production sector employs two input factors, capital $K_j(t)$ and labor force $N_j(t)$. Country $j{=}a,i,s$s production function $F_j(t)$ is specified as follows

$$F_i(t) = K_i^{\alpha_i} N_i^{\beta_i} G_i^{\zeta_i}, \ \alpha_i + \beta_i + \zeta_i = 1, \ \alpha_i, \beta_i, \zeta_i > 0.$$

(2)

where $A_j(t)$, $\alpha_j(t)$, and $\beta_j(t)$ are positive parameters. The profits are

$$F_i(t) = K_i^{\alpha_i} N_i^{\beta_i} G_i^{\zeta_i}, \ \alpha_i + \beta_i + \zeta_i = 1, \ \alpha_i, \beta_i, \zeta_i > 0.$$

The marginal conditions for maximizing the profits are

$$r = \frac{\alpha_i F_i}{K_i}, \quad w = \frac{\beta_i F_i}{N_i}, \quad p_s = \frac{\zeta_i F_i}{G_i}.$$

(3)

Current income and disposable income

The wage rate of group (j, q) is

$$w_{jq}(t) = w_j(t) H_{jq}^{m_{jq}(t)}(t), \quad (j, q) \in Q^*.$$

(4)

Per capita current income from the interest payment $r(t)\bar{k}_{jq}(t)$ and the wage payment $T_{jq}(t)w_{jq}(t)$ is

$$y_{jq}(t) = r(t)\bar{k}_{jq}(t) + T_{jq}(t)w_{jq}(t).$$

We call $y_{jq}(t)$ the current income in the sense that it comes from consumers' payment for human capital and efforts and consumers' current earnings from ownership of wealth. The total value of wealth that consumers can use is $\bar{k}_{jq}(t)$. Here, we assume that selling and buying wealth can be conducted instantaneously without any transaction cost. The per capita disposable income is given by

$$\hat{y}_{jq}(t) = y_{jq}(t) + \bar{k}_{jq}(t) = (1 + r(t))\bar{k}_{jq}(t) + W_{jq}(t).$$

(5)

where $W_{jq}(t) \equiv T_{jq}(t)w_{jq}(t)$ is the wage income.

Budgets and time constrains

The typical consumer distributes the total available budget between saving $s_{jq}(t)$, consumption of goods $c_{jq}(t)$. The budget constraint is

$$c_{jq}(t) + s_{jq}(t) = \hat{y}_{jq}(t) = (1 + r(t))\bar{k}_{jq}(t) + w_{jq}(t)T_{jq}(t),$$

(6)

The time constraint for everyone

$$T_{jq}(t) + \bar{T}_{jq}(t) = T_0,$$
(7)

where $\bar{T}_j(t)$ is the leisure time of the representative household and T_0 is the total available time. Insert (7) in (6)

$$w_{jq}(t)\bar{T}_{jq}(t) + c_{jq}(t) + s_{jq}(t) = \bar{y}_{jq}(t) \equiv (1 + r(t))\bar{k}_{jq}(t) + T_0\, w_{jq}(t).$$
(8)

Utility function and behavior of households

This study applies a utility function proposed by Zhang (1993). The variable $\bar{y}_{jq}(t)$ is the disposable income when the household spends all the available time on work. The representative consumer's utility function is specified as a function $\bar{T}_{jq}(t),\ C(t)$ and $K(t)+S(t)-\delta_k K(t)$ as follows

$$U_{jq}(t) = \bar{T}_{jq}^{\sigma_{jq0}(t)}(t)\, c^{\xi_{jq0}(t)}(t)\, s^{\lambda_{jq0}(t)}(t),\ \ \sigma_{jq0}(t),\ \xi_{jq0}(t),\ \lambda_{jq0}(t) > 0,$$
(9)

where $\sigma_{jq0}(t)$ is the propensity to use leisure time, $\xi_{jq0}(t)$ is the propensity to consume, and $\lambda_{jq0}(t)$ the propensity to own wealth. Maximizing U subject to (8) yields

$$T_h = \frac{\sigma\Omega}{wN},\ \ C = \xi\Omega,\ \ S = \lambda\Omega - (K - \delta_k)K$$
(10)

where

$$U(t) = T_h^{\sigma} C^{\xi} (K + S - \delta_k K)^{\lambda},\ \ \sigma, \xi,\ \lambda > 0$$

Change in the household wealth

According to the definitions of S_j the wealth accumulation of the representative household (j, q) is given by

$$\dot{\bar{k}}_{jq}(t) = s_{jq}(t) - \bar{k}_{jq}(t).$$

(11)

This equation simply states that the change in wealth is equal to saving minus dissaving.

Dynamics of human capital

This study applies Arrow's idea of "learning by producing" by Arrow (1962) to model human capital accumulation. first introduced learning by doing into growth theory. The basic idea is that people accumulate more skills and have more ideas when they are engaged in economic production. Following Zhang (2015), the human capital accumulation is specified as follows

$$K_1 + K_2 = K,$$

(12)

where $K_1 + K_2 = K,$ is the depreciation rates of human capital, $K_1 + K_2 = K,$ In (12), $\tilde{\upsilon}_{jq}(t)$, $a_{jq}(t)$, $\upsilon_{jq}(t)$, and $\theta_{jq}(t)$ are non-negative parameters, and $\pi_{jq}(t)$ is a parameter. The item $c_{jq}^{a_{jq}}$ implies a positive relation between human capital accumulation and consumption. The item $\bar{k}_{jq}^{\upsilon_{jq}}$ implies a positive relation between wealth and human capital accumulation. It can be interpreted that more wealth means, for instance, a higher social status. More wealth may also help one to maintain professional reputation. More work accumulates more human capital. The term $H_{jq}^{\pi_{jq}}$ implies that more human capital makes it easier (more difficult) to accumulate knowledge in the case of $\pi_{jq} < 0$ ($\pi_{jq} > 0$).

Demand of and supply for capital goods

The global capital employed is equal to the global wealth. This implies

$$K(t) = \sum_{j=1}^{J} K_j(t) = \sum_{j=1}^{J} \bar{K}_j(t) = \sum_{j=1}^{J} \sum_{q=1}^{Q_j} \bar{k}_{jq}(t) N_{jq},$$

(13)

where $\bar{K}_j(t)$ is the value of wealth owned by country j

$$\bar{K}_j(t) = \sum_{q=1}^{Q_j} \bar{k}_{jq}(t) N_{jq}.$$

(14)

The world production is equal to the world consumption and world net saving. That is

$$C(t) + S(t) - K(t) + \sum_{j=1}^{J} \delta_{kj} K_j(t) = F(t),$$

(15)

where

$$C(t) \equiv \sum_{j=1}^{J} \sum_{q=1}^{Q_j} c_{jq}(t) N_{jq}, \;\; S(t) \equiv \sum_{j=1}^{J} \sum_{q=1}^{Q_j} s_{jq}(t) N_{jq}, \;\; F(t) \equiv \sum_{j=1}^{J} F_j(t).$$

We completed the model. The model explains endogenous capital accumulation, human capital accumulation, labor supply, the international distribution of capital. The domestic markets of each country are perfectly competitive, international product and capital markets are freely mobile and labor is internationally immobile. The model is structurally general in the sense that some well-known models in theoretical economics can be considered as its special cases. For instance, if all the parameters are time-independent, the wealth and human capital are constant, the number of types of households is equal to the population, then the model is a Walrasian general equilibrium model. If all the parameters are time-independent and the population is homogeneous, our model is structurally similar to the neoclassical growth model by Solow (1956) and Uzawa (1961). It is structurally similar to the multi-class models by Pasinetti and Samuelson (e.g., Samuelson, 1959; Pasinetti, 1960, 1974) when all the parameters are time-independent.

3 The dynamics and its properties

As the system has many variables and these variables are nonlinearly interrelated with differential equations, consists of any number of types of households, its dynamics is highly dimensional. The following lemma shows a computational procedure to follow the motion of the economic system. We introduce a variable

$$z_1(t) \equiv \frac{r(t) + \delta_{k1}(t)}{w_1(t)}.$$

Lemma

The dynamics of the economy is governed by the following $2J$ dimensional differential equations system with $z(t)$, $\{\bar{k}_j(t)\}$, $(H_j(t))$, and t where $\{\bar{k}_j(t)\} \equiv (\bar{k}_2(t), \cdots, \bar{k}_J(t))$ and $(H_j(t)) \equiv (H_1(t), \cdots, H_J(t))$, as the variables

$$\dot{z}(t) = \Lambda_1\left(z(t), (H_j(t)), \{\bar{k}_j(t)\}, t\right),$$
$$\dot{\bar{k}}_j(t) = \Lambda_j\left(z(t), (H_j(t)), \{\bar{k}_j(t)\}, t\right), \quad j = 2, \ldots, J,$$
$$\dot{H}_j(t) = \Omega_j\left(z(t), (H_j(t)), \{\bar{k}_j(t)\}, t\right), \quad j = 1, \ldots, J,$$

in which Λ_j and Ω_j defined in the appendix are unique functions of $z(t)$, $\{\bar{k}_j(t)\}$, $(H_j(t))$, and t at any point in time. For given $z(t)$, $\{\bar{k}_j(t)\}$, and $(H_j(t))$, the other variables are uniquely determined at any point in time by the following procedure: $r(t)$ and $w(t)$ by (A3) $\rightarrow$ $w_j(t)$ by (A4) $\rightarrow$ $p(t)$ by (A5) $\rightarrow$ $\bar{k}_1(t)$ by (A18) $\rightarrow$ $N_i(t)$ by (A12) $\rightarrow$ $N(t)$ by (A11) $\rightarrow$ $N_s(t)$ by (A8) $\rightarrow$ $\bar{y}_j(t)$ by (A6) $\rightarrow$ $K_i(t)$ and $K_s(t)$ by (A1) $\rightarrow$ $F_i(t)$ and $F_s(t)$ by the definitions $\rightarrow$ $\bar{T}_j(t)$, $c_j(t)$, and $s_j(t)$ by (15) $\rightarrow$ $K(t)$ by (4).

Following the lemma, we can simulate the dynamic equations with any number of types of households. Following Zhang (2015), we first consider all the parameters time-independent. The rest of this section is based on Zhang's simulation results. We consider the world consists of three national economies, i.e., $J = 3$. The population and human capital utilization efficiency of the three economies are specified as follows

$$N_{j1} = 1, \; N_{j2} = 69, \; N_{j3} = 20, \; T_0 = 24, \quad j = 1, 2, 3,$$

$$\begin{pmatrix} m_{11} \\ m_{21} \\ m_{31} \end{pmatrix} = \begin{pmatrix} 0.8 \\ 0.7 \\ 0.65 \end{pmatrix}, \begin{pmatrix} m_{12} \\ m_{22} \\ m_{32} \end{pmatrix} = \begin{pmatrix} 0.2 \\ 0.15 \\ 0.18 \end{pmatrix}, \begin{pmatrix} m_{13} \\ m_{23} \\ m_{33} \end{pmatrix} = \begin{pmatrix} 0.15 \\ 0.18 \\ 0.15 \end{pmatrix}, \begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0.8 \\ 0.75 \end{pmatrix}, \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} = \begin{pmatrix} 0.31 \\ 0.3 \\ 0.3 \end{pmatrix}.$$

(16)

We specify the household preferences of the three economies as

$$\begin{pmatrix} \lambda_{110} \\ \lambda_{210} \\ \lambda_{310} \end{pmatrix} = \begin{pmatrix} 0.94 \\ 0.9 \\ 0.85 \end{pmatrix}, \begin{pmatrix} \sigma_{110} \\ \sigma_{210} \\ \sigma_{310} \end{pmatrix} = \begin{pmatrix} 0.4 \\ 0.25 \\ 0.25 \end{pmatrix}, \begin{pmatrix} \xi_{110} \\ \xi_{210} \\ \xi_{310} \end{pmatrix} = \begin{pmatrix} 0.07 \\ 0.07 \\ 0.07 \end{pmatrix}, \begin{pmatrix} \lambda_{120} \\ \lambda_{220} \\ \lambda_{320} \end{pmatrix} = \begin{pmatrix} 0.65 \\ 0.63 \\ 0.55 \end{pmatrix}, \begin{pmatrix} \sigma_{120} \\ \sigma_{220} \\ \sigma_{320} \end{pmatrix} = \begin{pmatrix} 0.2 \\ 0.2 \\ 0.2 \end{pmatrix},$$

$$\begin{pmatrix} \xi_{120} \\ \xi_{220} \\ \xi_{320} \end{pmatrix} = \begin{pmatrix} 0.18 \\ 0.18 \\ 0.18 \end{pmatrix}, \begin{pmatrix} \lambda_{130} \\ \lambda_{230} \\ \lambda_{330} \end{pmatrix} = \begin{pmatrix} 0.6 \\ 0.58 \\ 0.5 \end{pmatrix}, \begin{pmatrix} \sigma_{130} \\ \sigma_{230} \\ \sigma_{330} \end{pmatrix} = \begin{pmatrix} 0.2 \\ 0.2 \\ 0.2 \end{pmatrix}, \begin{pmatrix} \xi_{130} \\ \xi_{230} \\ \xi_{330} \end{pmatrix} = \begin{pmatrix} 0.2 \\ 0.2 \\ 0.2 \end{pmatrix}. \tag{17}$$

The parameters in the human capital accumulation equations are specified as follows

$$\begin{pmatrix} \tilde{v}_{11} \\ \tilde{v}_{12} \\ \tilde{v}_{13} \\ \tilde{v}_{21} \\ \tilde{v}_{22} \\ \tilde{v}_{23} \\ \tilde{v}_{31} \\ \tilde{v}_{32} \\ \tilde{v}_{33} \end{pmatrix} = \begin{pmatrix} 0.8 \\ 0.4 \\ 0.1 \\ 0.7 \\ 0.35 \\ 0.1 \\ 0.6 \\ 0.3 \\ 0.3 \end{pmatrix}, \begin{pmatrix} a_{11} \\ a_{12} \\ a_{13} \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{31} \\ a_{32} \\ a_{33} \end{pmatrix} = \begin{pmatrix} 0.3 \\ 0.2 \\ 0.1 \\ 0.3 \\ 0.2 \\ 0.1 \\ 0.3 \\ 0.2 \\ 0.2 \end{pmatrix}, \begin{pmatrix} v_{11} \\ v_{12} \\ v_{13} \\ v_{21} \\ v_{22} \\ v_{23} \\ v_{31} \\ v_{32} \\ v_{33} \end{pmatrix} = \begin{pmatrix} 0.2 \\ 0.1 \\ 0.1 \\ 0.2 \\ 0.1 \\ 0.1 \\ 0.2 \\ 0.1 \\ 0.1 \end{pmatrix}, \begin{pmatrix} \theta_{11} \\ \theta_{12} \\ \theta_{13} \\ \theta_{21} \\ \theta_{22} \\ \theta_{23} \\ \theta_{31} \\ \theta_{32} \\ \theta_{33} \end{pmatrix} = \begin{pmatrix} 0.2 \\ 0.1 \\ 0.1 \\ 0.2 \\ 0.1 \\ 0.1 \\ 0.2 \\ 0.1 \\ 0.1 \end{pmatrix}, \begin{pmatrix} \pi_{11} \\ \pi_{12} \\ \pi_{13} \\ \pi_{21} \\ \pi_{22} \\ \pi_{23} \\ \pi_{31} \\ \pi_{32} \\ \pi_{33} \end{pmatrix} = \begin{pmatrix} 0.1 \\ 0.2 \\ 0.4 \\ 0.12 \\ 0.2 \\ 0.4 \\ 0.15 \\ 0.2 \\ 0.2 \end{pmatrix}. \tag{18}$$

We specify

$$\delta_{j1} = 0.04, \quad \delta_{j2} = \delta_{j3} = 0.06, \quad j = 1, 2, 3,$$

We specify the initial conditions as follows

$z_1(0) = 0.059$, $\bar{k}_{12}(0) = 100$, $\bar{k}_{13}(0) = 60$, $\bar{k}_{21}(0) = 3860$, $\bar{k}_{22}(0) = 57$, $\bar{k}_{23}(0) = 42$, $\bar{k}_{31}(0) = 164$, $\bar{k}_{32}(0) = 47$, $\bar{k}_{33}(0) = 31$, $H_{11}(0) = 612$, $H_{12}(0) = 14$, $H_{13}(0) = 3$, $H_{21}(0) = 322$, $H_{22}(0) = 11$, $H_{23}(0) = 3.2$, $H_{31}(0) = 182$, $H_{32}(0) = 11$, $H_{33}(0) = 3$,

In Figure 1, the national output Y, the share of each group's wealth in the national wealth θ_{jw}, and the ratio between group 1's and another group's wealth φ_j, are respectively defined as

$$\varphi_j(t) \equiv \frac{\sum_{l=1}^{3} \bar{K}_{jl}(t)}{K(t)} 100, \quad \varphi_{jq}(t) \equiv \frac{\bar{K}_{jq}(t)}{\bar{K}_j(t)} 100, \quad \tilde{\varphi}_{jq}(t) \equiv \frac{\bar{k}_{j1}(t)}{\bar{k}_{jq}(t)} 100, \quad j = 1, 2, 3, \quad q = 1, 2, 3.$$