

A REMARKABLE PROPERTY OF EXCESSIVE FUNCTIONS CONE

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Abstract: In this paper we present remarkable properties of excessive functions with respect to absolute continue resolvent. The cone of above functions forms a H -cone.

Keywords: excessive function, H -cone, absolute continue resolvent

The excessive functions with respect to absolute continue resolvent.

Definiton 1. If (X, \mathcal{B}) is a measurable space and $p\mathcal{B}$ the numerical positive measurable functions on E we denote a kernel on (X, \mathcal{B}) a map $V: p\mathcal{B} \rightarrow p\mathcal{B}$ with properties:

1. $V0 = 0$
2. $V(\sum_n f_n) = \sum_n V(f_n)$.

Definition 2. A family $\mathcal{V} = (V_\alpha)_{\alpha > 0}$ of kernels on measurable space (X, \mathcal{B}) is called resolvent if the followings hold:

1. $V_\alpha V_\beta = V_\beta V_\alpha, \forall \alpha, \beta > 0$
2. $V_\alpha = V_\beta + (\beta - \alpha)V_\alpha V_\beta, \forall \alpha, \beta > 0, \alpha < \beta$.

The resolvent is called sub-Markovian if for any $\alpha > 0$ we have $\alpha V_\alpha \leq 1$.

The kernel $V = \sup_\alpha V_\alpha f$ is called initially kernel.

Definition 3. A map $s \in p\mathcal{B}$ is called \mathcal{V} - excessive if the followings hold:

1. s is \mathcal{V} -supermedian, i.e $\alpha V_\alpha s \leq s$ for any $\alpha > 0$
2. $\sup_\alpha \alpha V_\alpha s = s$
3. s is finite \mathcal{V} a.e. (a set $A \subset X$ is \mathcal{V} -negligible if there exists $A' \subset \mathcal{B}$ such that $A \subset A'$ and $V_\alpha(A') = 0$, for any $\alpha > 0$).

Proposition 1. We have the following properties:

1. If $s \in \mathcal{S}_\mathcal{V}$ it follows that $\hat{s} = \sup_\alpha V_\alpha s \in \mathcal{S}_\mathcal{V}, \hat{s} \leq s$
2. If $s, t \in \mathcal{S}_\mathcal{V}$ it follows that $\widehat{s+t} = \hat{s} + \hat{t}$
3. For any $(s_n)_n \subset \mathcal{S}_\mathcal{V}, s_n \uparrow s$ it follows that $s_n \uparrow s$
4. For any $s \in \mathcal{S}_\mathcal{V}$ it follows that $V_\alpha \hat{s} = V_\alpha s$
5. If $s \in \mathcal{S}_\mathcal{V} \Rightarrow \hat{s} = s$ \mathcal{V} - a.p.t.
6. If $s \in \mathcal{S}_\mathcal{V} \Rightarrow \hat{\hat{s}} = s$ see (e.g [1] Proposition 1.18)

Theorem 1. We have the following properties:

1. For any $t \in \mathcal{E}_\mathcal{V}$ (the set of excessive functions), $\alpha, \beta > 0$ it follows that $\alpha s + \beta t \in \mathcal{E}_\mathcal{V}$

2. For any $s, t \in \mathcal{E}_V, s \leq t$ \mathcal{V} -a. e. it follows that $s \leq t$
3. For any $(s_n)_n \subseteq \mathcal{E}_V$ there exists $\bigwedge_n s_n$ and we have $\bigwedge_n s_n = \overline{\inf} s_n$ and $s + \bigwedge_n s_n = \bigwedge_n (s + s_n)$, for any $s \in \mathcal{E}_V$
4. For any $(s_n)_n \subset \mathcal{E}_V$ dominated there exists $\bigvee_n s_n$ and we have $\bigvee_n s_n = R^V(\sup_n s_n)$, where $R^V f = \inf\{s \in \mathcal{S}_V \mid s \geq f\}$ ($s \in \mathcal{S}_V$ if and only if $\alpha \bigvee s \leq s$, for any $\alpha > 0$) (see e.g. [1] Theorem 1.1.9)
If $(s_n)_n$ is an increasing family we have $\bigvee_n s_n = \sup_n s_n$.
5. For any $s, t, u \in \mathcal{E}_V$ such that $s \leq t + u$, there exists $s', s'' \in \mathcal{E}_V$ such that $s' + s''$ and $s' \leq t, s'' \leq u$. (see e.g. [4] Theorem 1.1.9)

Definition 4. A resolvent $\mathcal{V} = (V_\alpha)_{\alpha > 0}$ is absolutely continue (with respect to a finite measure m) if for any $f \in \mathcal{F}$, such that $\int f dm = 0$ it follows that $V_\alpha f = 0$, for any $\alpha > 0$).

Theorem 2. Let $\mathcal{V} = (V_\alpha)_{\alpha > 0}$ an absolutely continue resolvent (with respect to a finite measure m). Then the followings hold:

1. For any increasingly and dominated family $(s_i)_{i \in I} \subset \mathcal{E}_V$ we have $\sup_{i \in I} s_i \in \mathcal{E}_V$ and there exists an increasing family $(s_{i_n})_{i_n \in I'}$, such that $\sup_n s_{i_n} = \sup_{i \in I} s_i = \bigvee_{i \in I} s_i$.
2. For any family $(s_i)_{i \in I} \in \mathcal{E}_V$ there exists a subsequence $(i_n)_n \subset I$, such that $\bigwedge_{i \in I} s_i = \overline{\inf} s_{i_n}$ and $s + \bigwedge_{i \in I} s_i = \bigwedge_{i \in I} (s + s_i)$, for any $s \in \mathcal{E}_V$ (see e.g [4] Theorem 1.1.10)

Definition 4. An ordered convex cone S is called H -cone if the following axioms are satisfied:

1. For any non-empty family $A \subset S$ there exists $\bigwedge A$ and we have $s + \bigwedge A = \bigwedge (s + A)$, for any $s \in S$;
2. For any increasing and dominated family $A \subset S$ there exists $\bigvee A$ and we have $\bigvee (A + u) = \bigvee A + u$, for any $u \in S$;
3. S satisfies the Riesz decomposition property i.e. for any $s, s_1, s_2 \in S$ such that $s \leq s_1 + s_2$ there exist $t_1, t_2 \in S$ satisfying $s = t_1 + t_2, t_1 \leq s_1, t_2 \leq s_2$.

From two theorem above it follows that the cone of excessive functions with respect to absolute continue resolvent forms a H -cone.

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