A REMARCABLE PROPERTY OF EXCESSIVE FUNCTIONS CONE

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Abstract: In this paper we present remarcable properties of excessive functions with respect to absolute continue resolvent. The cone of above functions forms a *H*-cone.

Keywords: excessive function, *H*-cone, absolute continue resolvent

The excessive functions with respect to absolute continue resolvent.

Definiton 1. If (X, \mathcal{B}) is a measurable space and $p\mathcal{B}$ the numerical positive measurable functions on E we denote a kernel on (X, \mathcal{B}) a map $V: p\mathcal{B} \to p\mathcal{B}$ with properties:

- 1. V0 = 0
- 2. $V(\sum_n f_n) = \sum_v V(f_n)$.

Definition 2. A family $V = (V_{\alpha})_{\alpha>0}$ of kernels on measurable space (X, β) is called resolvent if the followings hold:

- 1. $V_{\alpha}V_{\beta} = V_{\beta}V_{\alpha}, \forall \alpha, \beta > 0$
- 2. $V_{\alpha} = V_{\beta} + (\beta \alpha)V_{\alpha}V_{\beta}, \forall \alpha, \beta > 0, \alpha < \beta.$

The resolvent is called sub-Markovian if for any $\alpha > 0$ we have $\alpha V_{\alpha} \leq 1$. The kernel $V = \sup_{\alpha} V_{\alpha} f$ is called initially kernel.

Definition 3. A map $s \in p\mathcal{B}$ is called \mathcal{V} - excessive if the followings hold:

- 1. s is V –supermedian, i.e $\alpha V_{\alpha s} \le s$ for any $\alpha > 0$
- 2. $\sup_{\alpha} \alpha V_{\alpha} s = s$
- 3. s is finite V a.e. (a set $A \subseteq X$ is V-negligible if there exists $A' \subseteq B$ such that $A \subseteq A'$ and $V_{\alpha}(A') = 0$, for any $\alpha > 0$).

Proposition 1. We have the following properties:

- 1. If $s \in \mathcal{S}_{\mathcal{V}}$ it follows that $\hat{s} = \sup_{\alpha} V_{\alpha} s \in \mathcal{S}_{\mathcal{V}}$, $\hat{s} \leq s$
- 2. If $s, t \in S_v$ it follows that $\widehat{s+t} = \hat{s} + \hat{t}$
- 3. For any $(s_n)_n \subset S_v$, $s_n \uparrow s$ it follows that $s_n \uparrow s$
- 4. For any $s \in \mathcal{S}_{\mathcal{V}}$ it follows that $V_{\alpha}\hat{s} = V_{\alpha}s$
- 5. If $s \in \mathcal{S}_{\mathcal{V}} \Rightarrow \hat{s} = s \mathcal{V}$ a.p.t.
- 6. If $s \in \mathcal{S}_{v} \Rightarrow \hat{\hat{s}} = s$ see (e.g [1] Proposition 1.18)

Theorem 1. We have the following properties:

1. For any $t \in \mathcal{E}_{\mathcal{V}}$ (the set of excessive functions), $\alpha, \beta > 0$ it follows that $\alpha s + \beta t \in \mathcal{E}_{\mathcal{V}}$

- 2. For any $s, t \in \mathcal{E}_{v}$, $s \leq t \ \mathcal{V}$ -a. e. it follows that $s \leq t$
- 3. For any $(s_n)_n \subseteq \mathcal{E}_v$ there exists $\Lambda_n s_n$ and we have $\Lambda_n s_n = \inf s_n$ and $s + \Lambda_n s_n = \Lambda_n (s + s_n)$, for any $s \in \mathcal{E}_v$
- 4. For any $(s_n)_n \subset \mathcal{E}_{\mathcal{V}}$ dominated there exists $\bigvee_n s_n$ and we have $\bigvee_n s_n = R^{\mathcal{V}}(\sup_n s_n)$, where $R^{\mathcal{V}}f = \inf\{s \in \mathcal{S}_{\mathcal{V}} \mid s \geq f\}$ ($s \in \mathcal{S}_{\mathcal{V}}$ if and only if $\alpha \bigvee \alpha s \leq s$, for any $\alpha > 0$) (see e.g. [1] Theorem 1.1.9)

If $(s_n)_n$ is an increasing family we have $\bigvee_n s_n = \sup_n s_n$.

5. For any $s, t, u \in \mathcal{E}_{\mathcal{V}}$ such that $s \leq t + u$, there exists $s', s'' \in \mathcal{E}_{\mathcal{V}}$ such that s' + s'' and $s' \leq t, s'' \leq u$. (see e.g. [4] Theorem 1.1.9)

Definition 4. A resolvent $\mathcal{V} = (V_{\alpha})_{\alpha > 0}$ is absolutely continue (with respect to a finite measure m) if for any $f \in \mathcal{F}$, such that $\int f dm = 0$ it follows that $V_{\alpha}f = 0$, for any $\alpha > 0$).

Theorem 2. Let $\mathcal{V} = (V_{\alpha})_{\alpha > 0}$ an absolutely continue resolvent (with respect to a finite measure m). Then the followings hold:

- 1. For any increasingly and dominated family $(s_i)_{i \in I} \subset \mathcal{E}_{\mathcal{V}}$ we have $\sup_{i \in I} s_i \in \mathcal{E}_{\mathcal{V}}$ and there exists an increasing family $(s_{i_n})_{i_n \in I'}$ such that
- $\sup_{n} s_{i_n} = \sup_{i \in I} s_i = \bigvee_{i \in I} s_i.$
- 2. For any family $(s_i)_{i \in I} \in \mathcal{E}_{\mathcal{V}}$ there exists a subsequence $(i_n)_n \subset I$, such that $\bigwedge_{i \in I} s_i = \inf s_{i_n}$ and $s + \bigwedge_{i \in I} s_i = \bigwedge_{i \in I} (s + s_i)$, for any $s \in \mathcal{E}_{\mathcal{V}}$ (see e.g [4] Theorem 1.1.10)

Definition 4. An ordered convex cone **5** is called **H**-cone if the following axioms are satisfied:

- 1. For any non-empty family $A \subseteq S$ there exists $\bigwedge A$ and we have $s + \bigwedge A = \bigwedge (s + A)$, for any $s \in S$;
- 2. For any increasing and dominated family $A \subseteq S$ there exists VA and we have V(A = u) = VA + u, for any $u \in S$;
- 3. S satisfies the Riesz decomposition property i.e. for any $s, s_1, s_2 \in S$ such that $s \leq s_1 + s_2$ there exist $t_1, t_2 \in S$ satisfying $s = t_1 + t_2$, $t_1 \leq s_1, t_2 \leq s_2$.

From two theorem above it follows that the cone of excessive functions with respect to absolute continue resolvent forms a *H*-cone.

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