A REMARCABLE PROPERTY OF EXCESSIVE FUNCTIONS CONE

Diana Mărginean Petrovai, Assistant Professor Phd,
"Petru Maior" University of Tîrgu Mureş

Abstract: In this paper we present remarcable properties of excessive functions with respect to absolute continue resolvent. The cone of above functions forms a H-cone.

Keywords: excessive function, H-cone, absolute continue resolvent

The excessive functions with respect to absolute continue resolvent.

Definiton 1. If $(X,\mathcal{B})$ is a measurable space and $p\mathcal{B}$ the numerical positive measurable functions on E we denote a kernel on $(X,\mathcal{B})$ a map $V: p\mathcal{B} \to p\mathcal{B}$ with properties:

1. $V0 = 0$
2. $V(\sum_n f_n) = \sum_v V(f_n)$.

Definition 2. A family $\mathcal{V} = (V_\alpha)_{\alpha>0}$ of kernels on measurable space (X,β) is called resolvent if the followings hold:

1. $V_\alpha V_\beta = V_\beta V_\alpha, \ \forall \alpha,\beta > 0$
2. $V_\alpha = V_\beta + (\beta-\alpha)V_\alpha V_\beta, \forall \alpha,\beta > 0, \alpha < \beta.$

The resolvent is called sub-Markovian if for any $\alpha > 0$ we have $\alpha V_\alpha \leq 1.$
The kernel $V = \sup_\alpha V_\alpha f$ is called initially kernel.

Definition 3. A map $s \in p\mathcal{B}$ is called $\mathcal{V}$- excessive if the followings hold:

1. s is $\mathcal{V}$ –supermedian, i.e $\alpha V_{\alpha s} \leq s$ for any $\alpha > 0$
2. $\sup_\alpha \alpha V_\alpha s = s$
3. s is finite $\mathcal{V}$ a.e. (a set $A \subset X$ is $\mathcal{V}$-negligible if there exists $A' \subset \mathcal{B}$ such that $A \subset A'$ and $V_\alpha(A') = 0$, for any $\alpha > 0$).

Proposition 1. We have the following properties:

1. If $s \in \mathcal{S}_\mathcal{V}$ it follows that $\hat{s} = \sup_\alpha V_\alpha s \in \mathcal{S}_\mathcal{V}, \ \hat{s} \leq s$
2. If $s,t \in \mathcal{S}_\mathcal{V}$ it follows that $\widehat{s+t} = \hat{s} + \hat{t}$
3. For any $(s_n)_n \subset \mathcal{S}_\mathcal{V}$, $s_n \uparrow s$ it follows that $s_n \uparrow s$
4. For any $s \in \mathcal{S}_\mathcal{V}$ it follows that $V_\alpha \hat{s} = V_\alpha s$
5. If $s \in \mathcal{S}_\mathcal{V} \Rightarrow \hat{s} = s$ $\mathcal{V}$- a.p.t.
6. If $s \in \mathcal{S}_\mathcal{V} \Rightarrow \hat{\hat{s}} = s$ see (e.g [1] Proposition 1.18)

Theorem 1. We have the following properties:

1. For any $t \in \mathcal{E}_\mathcal{V}$ (the set of excessive functions), $\alpha,\beta > 0$ it follows that $\alpha s + \beta t \in \mathcal{E}_\mathcal{V}$

2. For any $s, t \in \mathcal{E}_{\mathcal{V}}, s \leq t$ $\mathcal{V}$-a. e. it follows that $s \leq t$
3. For any $(s_n)_n \subseteq \mathcal{E}_v$ there exists $\bigwedge_n s_n$ and we have $\bigwedge_n s_n = \widetilde{\inf} s_n$ and $s + \bigwedge_n s_n = \bigwedge_n(s + s_n)$, for any $s \in \mathcal{E}_{\mathcal{V}}$
4. For any $(s_n)_n \subset \mathcal{E}_{\mathcal{V}}$ dominated there exists $\bigvee_n s_n$ and we have $\bigvee_n s_n = R^{\mathcal{V}}(\sup_n s_n)$, where $R^{\mathcal{V}} f = \inf\{s \in \mathcal{S}_{\mathcal{V}} \mid s \geq f\}$ ($s \in \mathcal{S}_{\mathcal{V}}$ if and only if $\alpha V \alpha s \leq s$, for any $\alpha > 0$) (see e.g. [1] Theorem 1.1.9)

If $(s_n)_n$ is an increasing family we have $\bigvee_n s_n = \sup_n s_n$.

5. For any $s, t, u \in \mathcal{E}_{\mathcal{V}}$ such that $s \leq t + u$, there exists $s', s'' \in \mathcal{E}_{\mathcal{V}}$ such that $s' + s''$ and $s' \leq t, s'' \leq u$. (see e.g. [4] Theorem 1.1.9)

Definition 4. A resolvent $\mathcal{V} = (V_\alpha)_{\alpha > 0}$ is absolutely continue (with respect to a finite measure m) if for any $f \in \mathcal{F}$, such that $\int f dm = 0$ it follows that $V_\alpha f = 0$, for any $\alpha > 0$).

Theorem 2. Let $\mathcal{V} = (V_\alpha)_{\alpha > 0}$ an absolutely continue resolvent (with respect to a finite measure m). Then the followings hold:

1. For any increasingly and dominated family $(s_i)_{i \in I} \subset \mathcal{E}_{\mathcal{V}}$ we have $\sup_{i \in I} s_i \in \mathcal{E}_{\mathcal{V}}$ and there exists an increasing family $(s_{i_n})_{i_n \in I}$, such that $\sup_n s_{i_n} = \sup_{i \in I} s_i = \bigvee_{i \in I} s_i$.
2. For any family $(s_i)_{i \in I} \in \mathcal{E}_{\mathcal{V}}$ there exists a subsequence $(i_n)_n \subset I$, such that $\bigwedge_{i \in I} s_i = \widetilde{\inf} s_{i_n}$ and $s + \bigwedge_{i \in I} s_i = \bigwedge_{i \in I}(s + s_i)$, for any $s \in \mathcal{E}_{\mathcal{V}}$ (see e.g [4] Theorem 1.1.10)

Definition 4. An ordered convex cone S is called H-cone if the following axioms are satisfied:

1. For any non-empty family $A \subset S$ there exists $\bigwedge A$ and we have $s + \bigwedge A = \bigwedge(s + A)$, for any $s \in S$;
2. For any increasing and dominated family $A \subset S$ there exists $\bigvee A$ and we have $\bigvee(A = u) = \bigvee A + u$, for any $u \in S$;
3. S satisfies the Riesz decomposition property i.e. for any $s, s_1, s_2 \in S$ such that $s \leq s_1 + s_2$ there exist $t_1, t_2 \in S$ satisfying $s = t_1 + t_2$, $t_1 \leq s_1, t_2 \leq s_2$.

From two theorem above it follows that the cone of excessive functions with respect to absolute continue resolvent forms a H-cone.

References

1. N. Boboc, Gh. Bucur, Order and Convexity in Potential Theory: H-cones, Lecture Notes in Math. 853, Springer-Verlag 1981.
2. M. Șabac, Lecții de analizț reală. Capitole de teoria măsurii și integralei, Editura București, 1982.
3. N. Boboc, Gh. Bucur, Măsură și capacitate, Editura științifică și enciclopedică, București, 1985.

4. N. Boboc, Analizăa matematică, Editura Fundamentum, București, 1998.
5. W. Rudin, Principles of Mathematical Analysis, Mc Grow-Hill, New-York, 1964.
6. N. Boboc, Gh. Bucur, Potentials and supermedian function on fine open sets in standard H-cones, Preprint Series in Mathematics No. 59, 1984, București.
7. N. Boboc, Gh. Bucur, A. Cornea, Natural Topologies on H-Cones. Weak Completeness, Preprint Series in Mathematics No. 12, 1978, București.
8. N. Boboc, Gh. Bucur, A. Cornea, Carrier Theory and Negligible Sets on a Standard H-Cone of Functions, Preprint Series in Mathematics No. 25, 1978, București.
9. N. Boboc, Gh. Bucur, Potentials in standard H-cones of functions, Preprint Series in Mathematics No. 6, 1988, București.
10. N. Boboc, Gh. Bucur, Potential in Standard H-Cones, Preprint Series in Mathematics No. 61, 1979, București.
11. R. Cristescu, Analiză funcțională, Editura didactică și pedagogică, București, 1979.
12. D. Mărginean Petrovai, New Properties of Excessive Measures, Mathematical Reports, vol 10 (60), no. 4, 2008
13. D. Mărginean Petrovai, Positive Measures and Outer Measures on a σ - Algebra of Sets, Interdisciplinarity in Engineering Proceedings, 2012, 382-384.
14. D. Mărginean Petrovai, The Most Important Outer Measures, Interdisciplinarity in Engineering Proceedings, 2012, 385-387.
15. D. Mărginean Petrovai, Lebesgue-Stieltjes measure on $\mathbb{R}$, Elsevier, Procedia Tehnology, vol. 12, 2014, 234--239.